

# Moving Beyond the Average Treatment Effect: A Look at Power Analyses for Moderator Effects in Cluster Randomized Trials

Jessaca Spybrook

Western Michigan University

December 3, 2015

(Joint work with Ben Kelcey, University of Cincinnati, Nianbo Dong, University of Missouri)

\*This work was funded by a grant (DGE-1437692) from the National Science Foundation.

- Background
- Purpose
- 2-level CRT
- 3-level CRT
- Implications

# ■ What works?

- What works?
- Under what conditions?

- What works?
- Under what conditions?
- For whom?

- What works?
- *Under what conditions?*
- *For whom?*

## ■ Designs

- 2-level cluster randomized trial  
(2-level CRT)
  
- 3-level cluster randomized trial  
(3-level CRT)

- Suppose a team of researchers are planning an impact study to determine the effectiveness of a new 5<sup>th</sup> grade science curriculum. They are planning a 2-level CRT with students nested within schools and schools are randomly assigned to the new curriculum or the current curriculum.



## ■ *What works?*

What is the effect of the new science curriculum relative to the current curriculum on science achievement for fifth graders?

## ■ Model

$$y_{ij} = \gamma_{00} + \gamma_{01}T_j + r_{0j} + e_{ij}$$

$$e_{ij} \sim N(0, \sigma^2)$$

$$r_{0j} \sim N(0, \tau)$$

Main effect of Treatment



## ■ Model

$$y_{ij} = \gamma_{00} + \gamma_{01}T_j + r_{0j} + e_{ij}$$

$$e_{ij} \sim N(0, \sigma^2)$$

$$r_{0j} \sim N(0, \tau)$$

Main effect of Treatment



Set

$$\rho = \frac{\tau}{\tau + \sigma^2} \quad \delta = \frac{\gamma_{01}}{\sqrt{\tau + \sigma^2}}$$

- Minimum detectable effect size (MDES) (Bloom, 1995)

$$MDES = M_{J-2} \sqrt{\frac{4(\rho + (1 - \rho)/n)}{J}}$$

$M_{J-2}$  is multiplier that approaches 2.8 as J increases, two-tail test with 0.05 level of significance

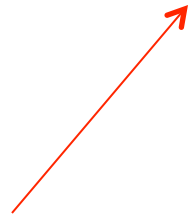
$J$  is total number of sites

$n$  is number of individuals per site

$\rho$  is the intraclass correlation

- To increase precision
  - Common practice to include a cluster-level covariate
  - School-level pretest

■ Model:  $y_{ij} = \gamma_{00} + \gamma_{01}T_j + \gamma_{02}W_j + r_{0j} + e_{ij}$



Main effect of Treatment

**Question****Effect of Interest****MDES**

What works?

Main effect of treatment

$$M_{J-2} \sqrt{\frac{4(\rho + (1 - \rho)/n)}{J}}$$

Main effect of treatment w/ covariate

$$M_{J-3} \sqrt{\frac{4((1 - R_{|W}^2)\rho + (1 - \rho)/n)}{J}}$$

# 2-level CRT – Minimum Detectable Effect Size

---

## What Works?

### *Main Effect*

---

J=30

n=10                      0.39

n=50                      0.29

n=100                     0.27

J=60

n=10                      0.27

n=50                      0.20

n=100                     0.19

---

Assumptions: Two-tail significance = 0.05, equal allocation of clusters, constant  $n$  per cluster, power = 0.80,  $\rho = 0.23$ ,  $R_{|W}^2 = 0.75$ .

## ■ *Under what conditions?*

Is there a difference in the mean effects by school type (urban vs. suburban)?



■ **Model:**

$$y_{ij} = \gamma_{00} + \gamma_{01}T_j + \gamma_{02}S_j + \gamma_{03}T_jS_j + \gamma_{04}W_j + r_{0j} + e_{ij}$$

Cluster level  
moderator effect



**Question****Effect of Interest****MDES  
MDES<sub>D</sub>**

What works?

Main effect of treatment

$$M_{J-2} \sqrt{\frac{4(\rho + (1 - \rho)/n)}{J}}$$

Main effect of treatment w/ covariate

$$M_{J-3} \sqrt{\frac{4((1 - R_{|W}^2)\rho + (1 - \rho)/n)}{J}}$$

Under what conditions?

Cluster-level moderator

$$M_{J-5} \sqrt{\frac{16((1 - R_{|SW}^2)\rho + (1 - \rho)/n)}{J}}$$

## 2-level CRT – Minimum Detectable Effect Size

	What Works? <i>Main Effect</i>	Under what conditions? <i>CL Mod</i>
J=30		
n=10	0.39	0.75
n=50	0.29	0.53
n=100	0.27	0.49
J=60		
n=10	0.27	0.52
n=50	0.20	0.37
n=100	0.19	0.34

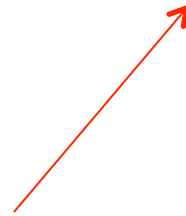
Assumptions: Two-tail significance = 0.05, equal allocation of clusters, constant  $n$  per cluster, power = 0.80,  $\rho = 0.23$ ,  $R_{|W}^2 = 0.75$ ,  $R_{|SW}^2 = 0.80$ .

■ ***For whom?***

Is there a difference in the mean effect for boys and girls?

■ Model:

$$y_{ij} = \gamma_{00} + \gamma_{01}T_j + \gamma_{10}X_{ij} + \gamma_{11}T_jX_{ij} + r_{0j} + e_{ij}$$



Individual level  
moderator effect

**Question****Effect of Interest****MDES  
MDES<sub>D</sub>**

What works?

Main effect of treatment

$$M_{J-2} \sqrt{\frac{4(\rho + (1 - \rho)/n)}{J}}$$

Main effect of treatment w/ covariate

$$M_{J-3} \sqrt{\frac{4((1 - R_{|W}^2)\rho + (1 - \rho)/n)}{J}}$$

Under what conditions?

Cluster-level moderator

$$M_{J-5} \sqrt{\frac{16((1 - R_{|SW}^2)\rho + (1 - \rho)/n)}{J}}$$

For whom?

Individual-level moderator

$$M_{n*J-(J-2)} \sqrt{\frac{16((1 - R_{|X}^2)(1 - \rho)/n)}{J}}$$

## 2-level CRT – Minimum Detectable Effect Size

	What Works? <i>Main Effect</i>	Under what conditions? <i>CL Mod</i>	For whom? <i>Ind Mod</i>
J=30			
n=10	0.39	0.75	0.51
n=50	0.29	0.53	0.23
n=100	0.27	0.49	0.16
J=60			
n=10	0.27	0.52	0.36
n=50	0.20	0.37	0.16
n=100	0.19	0.34	0.11

Assumptions: Two-tail significance = 0.05, equal allocation of clusters, constant  $n$  per cluster, power = 0.80,  $\rho = 0.23$ ,  $R_{|W}^2 = 0.75$ ,  $R_{|SW}^2 = 0.80$ , and  $R_{|X}^2 = 0.20$ .

- Suppose a team of researchers are planning an impact study to determine the effectiveness of a school-wide science curriculum. They are planning a 3-level CRT with students nested within teachers nested within schools and schools are randomly assigned to the new curriculum or the current curriculum.



## ■ *What works?*

What is the effect of the new science curriculum relative to the current curriculum on science achievement for all students?

## ■ *Under what conditions?*

Is there a difference in the mean effects by school type (urban vs. suburban)?

Is there a difference in the mean effects by teacher experience level (0-3 years vs. more than 3 years)?

■ ***For whom?***

Is there a difference in the mean effect for boys and girls?

Question	Effect of Interest	MDES
----------	--------------------	------

What works?	Main effect of treatment	$M_{K-3} \sqrt{\frac{4 \left\{ (1 - R_{ W}^2) \rho_{L3} + [\rho_{L2} + (1 - \rho_{L3} - \rho_{L2}) / n] / J \right\}}{K}}$
-------------	--------------------------	--

**Question**   **Effect of Interest**

**MDES**  
**MDES<sub>D</sub>**

What works?   Main effect of treatment    $M_{K-3} \sqrt{\frac{4 \left\{ (1 - R_{|W}^2) \rho_{L3} + [\rho_{L2} + (1 - \rho_{L3} - \rho_{L2}) / n] / J \right\}}{K}}$

Under what conditions?   Cluster-level moderator    $M_{K-5} \sqrt{\frac{16 \left\{ (1 - R_{|SW}^2) \rho_{L3} + [\rho_{L2} + (1 - \rho_{L3} - \rho_{L2}) / n] / J \right\}}{K}}$

Question	Effect of Interest		MDES MDES <sub>D</sub>
What works?	Main effect of treatment	$M_{K-3}$	$\sqrt{\frac{4\left\{\left(1 - R_{ W}^2\right)\rho_{L3} + \left[\rho_{L2} + \left(1 - \rho_{L3} - \rho_{L2}\right) / n\right] / J\right\}}{K}}$
Under what conditions?	Cluster-level moderator	$M_{K-5}$	$\sqrt{\frac{16\left\{\left(1 - R_{ SW}^2\right)\rho_{L3} + \left[\rho_{L2} + \left(1 - \rho_{L3} - \rho_{L2}\right) / n\right] / J\right\}}{K}}$
Under what conditions?	Teacher-level moderator	$M_u$	$\sqrt{\frac{16\left\{\left(1 - R_{ M}^2\right)\rho_{L2} + \left(1 - \rho_{L3} - \rho_{L2}\right) / n\right\} / J}{K}}$

Question	Effect of Interest	MDES MDES <sub>D</sub>
What works?	Main effect of treatment	$M_{K-3} \sqrt{\frac{4 \left\{ (1 - R_{ W}^2) \rho_{L3} + [\rho_{L2} + (1 - \rho_{L3} - \rho_{L2}) / n] / J \right\}}{K}}$
Under what conditions?	Cluster-level moderator	$M_{K-5} \sqrt{\frac{16 \left\{ (1 - R_{ SW}^2) \rho_{L3} + [\rho_{L2} + (1 - \rho_{L3} - \rho_{L2}) / n] / J \right\}}{K}}$
Under what conditions?	Teacher-level moderator	$M_u \sqrt{\frac{16 \left\{ (1 - R_{ M}^2) \rho_{L2} + (1 - \rho_{L3} - \rho_{L2}) / n \right\} / J}{K}}$
For whom?	Individual-level moderator	$M_v \sqrt{\frac{16 \left\{ (1 - R_{ X}^2) (1 - \rho_{L3} - \rho_{L2}) / n \right\} / J}{K}}$

Note:  $u = J * K - (K - 2)$ ,  $v = n * J * K - (J * K) - (K - 2)$

# 3-level CRT – Minimum Detectable Effect Size

---

## What Works? *Main Effect*

---

J=5

n=10                      0.24

n=30                      0.22

J=30

n=10                      0.19

n=30                      0.18

---

Assumptions: Two-tail significance = 0.05, equal allocation of clusters, constant J per cluster, constant n per teacher, 40 schools,

$\rho_{L3} = 0.15$ ,  $\rho_{L2} = 0.08$ ,  $R_{|W}^2 = 0.75$ ,  $R_{|S}^2 = 0.80$ ,  $R_{|P}^2 = 0.30$ , and  $R_{|X}^2 = 0.20$



## 3-level CRT – Minimum Detectable Effect Size

---

	<b>What Works? <i>Main Effect</i></b>	<b>Under what conditions? <i>CL Mod</i></b>
J=5		
n=10	0.24	0.45
n=30	0.22	0.41
J=30		
n=10	0.19	0.34
n=30	0.18	0.33

---

Assumptions: Two-tail significance = 0.05, equal allocation of clusters, constant J per cluster, constant  $n$  per teacher, 40 schools,  $\rho_{L3} = 0.15$ ,  $\rho_{L2} = 0.08$ ,  $R_{|W}^2 = 0.75$ ,  $R_{|S}^2 = 0.80$ ,  $R_{|P}^2 = 0.30$ , and  $R_{|X}^2 = 0.20$

## 3-level CRT – Minimum Detectable Effect Size

	<b>What Works? <i>Main Effect</i></b>	<b>Under what conditions? <i>CL Mod</i></b>	<b>Under what conditions? <i>TL Mod</i></b>
<b>J=5</b>			
n=10	0.24	0.45	0.29
n=30	0.22	0.41	0.23
<b>J=30</b>			
n=10	0.19	0.34	0.12
n=30	0.18	0.33	0.09

Assumptions: Two-tail significance = 0.05, equal allocation of clusters, constant J per cluster, constant n per teacher, 40 schools,

$\rho_{L3} = 0.15$ ,  $\rho_{L2} = 0.08$ ,  $R_{|W}^2 = 0.75$ ,  $R_{|S}^2 = 0.80$ ,  $R_{|P}^2 = 0.30$ , and  $R_{|X}^2 = 0.20$

## 3-level CRT – Minimum Detectable Effect Size

	<b>What Works?</b> <i>Main Effect</i>	<b>Under what conditions?</b> <i>CL Mod</i>	<b>Under what conditions?</b> <i>TL Mod</i>	<b>For whom?</b> <i>Ind Mod</i>
<b>J=5</b>				
n=10	0.24	0.45	0.29	0.18
n=30	0.22	0.41	0.23	0.11
<b>J=30</b>				
n=10	0.19	0.34	0.12	0.08
n=30	0.18	0.33	0.09	0.04

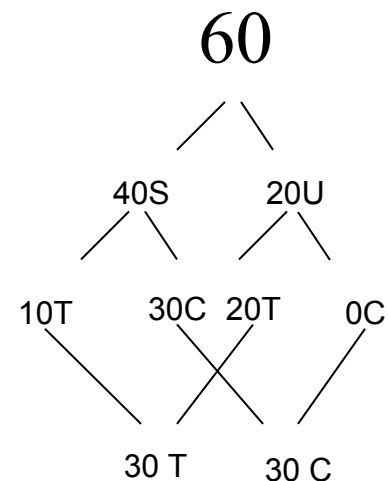
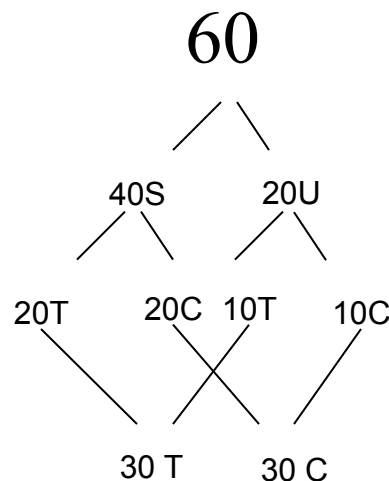
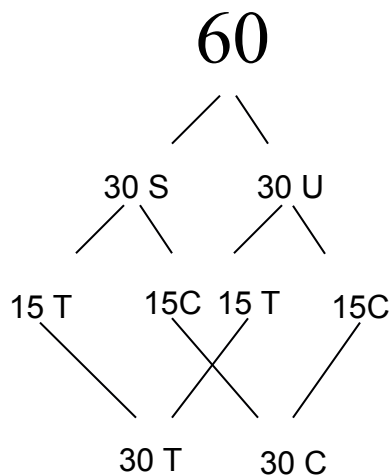
Assumptions: Two-tail significance = 0.05, equal allocation of clusters, constant J per cluster, constant n per teacher, 40 schools,

$\rho_{L3} = 0.15$ ,  $\rho_{L2} = 0.08$ ,  $R_{|W}^2 = 0.75$ ,  $R_{|S}^2 = 0.80$ ,  $R_{|P}^2 = 0.30$ , and  $R_{|X}^2 = 0.20$

## ■ *Under what conditions?*

### □ Cluster level moderator

- Challenging given current study sizes
- Effectiveness studies rather than efficacy studies
- If priority, need to consider in design stage



■ ***Under what conditions?***

□ Teacher level moderator

- Number of teachers per school matters
- If priority, need to consider in design phase

■ ***For whom?***

□ Individual level moderator

- In many cases, reasonable given current study sizes
- If priority, need to consider in design phase

## ■ Extensions

- Multisite studies
- R code available – [jessaca.spybrook@wmich.edu](mailto:jessaca.spybrook@wmich.edu)
- Some parts implemented in PowerUP! - <http://web.missouri.edu/~dongn/>
- Add to Optimal Design Plus



Thank You!

Questions?

Please email me: [jessaca.spybrook@wmich.edu](mailto:jessaca.spybrook@wmich.edu)