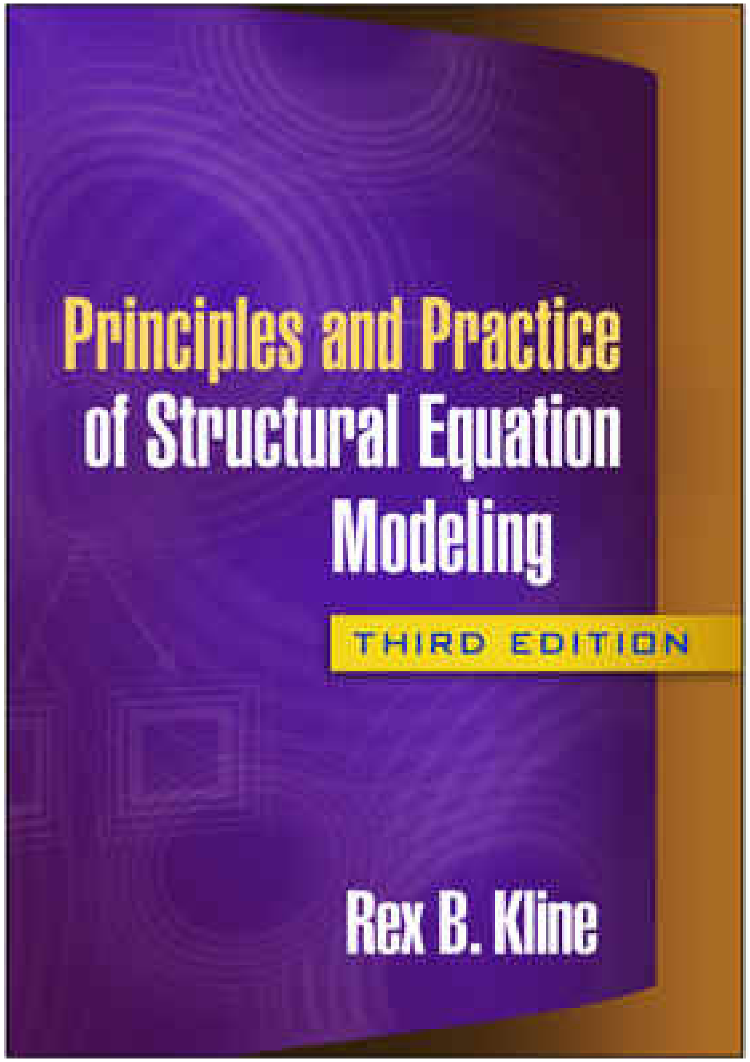
New developments in structural equation modeling

Rex B Kline
Concordia University
Montréal

Set A: SCM

UNL Methodology Workshop


Welcome


Principles and Practice
of Structural Equation
Modeling
THIRD EDITION
Rex B. Kline

A3


BACK
SPACE

Topics

- Graph theory
- Mediation:

  Design
  Conditional
  Causal

Topics

- Graph theory:

    Pearl's SCM

    Causal reasoning

    Causal estimation

Topics

- Mediation:

  Design requirements

  Conditional process modeling

  Cause × mediator (SCM)

Graph theory

- Pearl, J. (2009a). Causal inference in statistics: An overview. *Statistics Surveys*, *3*, 96–146.
- Pearl, J. (2009b). *Causality: Models, reasoning, and inference* (2nd ed.). New York: Cambridge University Press.
- Pearl, J. (2012). The causal foundations of structural equation modeling. In R. H. Hoyle (Ed.), *Handbook of structural equation modeling* (pp. 68–91). New York: Guilford Press.

Graph theory

- Bollen, K. A., & Pearl, J. (2013). Eight myths about causality and structural equation models. In S.L. Morgan (Ed.), *Handbook of causal analysis for social research* (pp. 301–328). New York: Springer.
- Cole, S. R., Platt, R. W., Schisterman, E. F., Chu, H., Westreich, D., Richardson, D., & Poole, C. (2010). Illustrating bias due to conditioning on a collider. *International Journal of Epidemiology*, *39*, 417–420
- Elwert, F. (2013). Graphical causal models. In S. L. Morgan (Ed.), *Handbook of causal analysis for social research* (pp. 245–273). New York, NY: Springer.

Graph theory

- Elwert, F. (2014). Endogenous selection bias: The problem of conditioning on a collider variable. *Annual Review of Sociology*, *40*, 31–53.
- Glymour, M. M. (2006). Using causal diagrams to understand common problems in social epidemiology. In M. Oakes & J. Kaufman (Eds), *Methods in social epidemiology* (pp. 387–422). San Francisco: Jossey-Bass.
- Hayduk, L., Cummings, G., Stratkotter, R., Nimmo, M., Grygoryev, K., Dosman, D., … Boadu, K. (2003). Pearl's d-separation: One more step into causal thinking. *Structural Equation Modeling*, *10*, 289–311.

Graph theory

- Kenny, D. A. (2014). Mediation. Retrieved from http://davidakenny.net/cm/mediate.htm#CI
- Shipley, B. (2000). A new inferential test for path models based on directed acyclic graphs. *Structural Equation Modeling*, *7*, 206–218.
- Spector, P. E., & Brannick, M. T. (2011). Methodological urban legends: The misuse of statistical control variables. *Organizational Research Methods*, *14*, 287–305.

Graph theory

- Knüppel, S., & Stang, A. (2010). DAG Program: Identifying minimal sufficient adjustment sets. *Epidemiology*, *21*, 159. http://epi.dife.de/dag/
- Porter, K., Poole, D., Kisynski, J., Sueda, S., & Knoll, B., Mackworth, A., … Hoos, H., Gorniak, P., & Conati, C. (1999–2009). Belief and Decision Network Tool (Version 5.1.10) (computer software). http://aispace.org/bayes/
- Textor, J., Hardt, J., & Knüppel, S. (2011). DAGitty: A graphical tool for analyzing causal diagrams. *Epidemiology*, *5*, 745. http://www.dagitty.net/

Mediation

- Design:

Cheung, J., & MacKinnon, D. P. (2012). Mediational/indirect effects in structural equation modeling. In R. H. Hoyle (Ed.), *Handbook of structural equation modeling* (pp. 417–435). New York: Guilford Press.

Little, T. D. (2013). *Longitudinal structural equation modeling*. New York, NY: Guilford.

MacKinnon, D. P. (2011). Integrating mediators and moderators in research design. *Research on Social Work Practice*, *21*, 675–681.

Mediation

- Design:

Maxwell, S. E., & Cole, D. A. (2007). Bias in cross-sectional analyses of longitudinal mediation. *Psychological Methods*, *12*, 23–44.

Selig, J. P., & Preacher, K. J. (2009). Mediation models for longitudinal data in developmental research. *Research in Human Development*, *6*, 144–164.

Wu, A. D., & Zumbo, B. D. (2008). Understanding and using mediators and moderators. *Social Indicators Research*, *87*, 367–392.

Mediation

- Conditional:

Edwards, J. R., & Lambert, L. S. (2007). Methods for integrating moderation and mediation: A general analytical framework using moderated path analysis. *Psychological Methods*, *12*, 1–22.

Hayes, A. F. (2013). *Introduction to mediation, moderation, and process control analysis: A regression-based approach*. New York: Guilford Press.

Mediation

- Conditional:

Hayes, A. F., & Preacher, K. J. (2013). Conditional process modeling: Using structural equation modeling to examine contingent causal processes, In G. R. Hancock & R. O. Mueller (Eds.), *Structural equation modeling: A second course* (pp. 219–266). Charlotte: IAP.

Preacher, K. J., Rucker, D. D., & Hayes, A. F. (2007). Addressing moderated mediation hypotheses: Theory, methods, and prescriptions. *Multivariate Behavioral Research*, *42*, 185–227.

Mediation

- Causal:

Bullock, J. G., Green, D. P., & Ha, S. E. (2010). Yes, but what's the mechanism? (Don't expect an easy answer). *Journal of Personality and Social Psychology*, *98*, 550–558.

Imai, K., Keele, L., & Yamamoto, T. (2010) Identification, inference and sensitivity analysis for causal mediation effects. *Statistical Science*, *25*, 51–71.

Mediation

- Causal:

Lange, T., Vansteelandt, S., & Bekaert, M. 2012). A simple unified approach for estimating natural direct and indirect effects. *American Journal of Epidemiology*, *176*, 190–195.

Pearl, J. (2014). Interpretation and identification of causal mediation. *Psychological Methods*. Advance online publication. http://dx.doi.org/10.1037/a0036434

Mediation

- Causal:

Petersen, M. L., Sinisi, S. E., & van der Laan, M. J. (2006). Estimation of direct causal effects. *Epidemiology*, *17*, 276–284.

Valeri, L., & VanderWeele, T. J. (2013). Mediation analysis allowing for exposure–mediator interactions and causal Interpretation: Theoretical assumptions and implementation with SAS and SPSS macros. *Psychological Methods*, *18*, 137–150.

Mediation

- Hicks, R., & Tingley, D. H. (2012). MEDIATION: Stata module for causal mediation analysis and sensitivity analysis [computer software]. http://EconPapers.repec.org/RePEc:boc:bocode:s457294
- Muthén, B. O. (2011). Applications of causally defined direct and indirect effects in mediation analysis using SEM in Mplus. https://www.statmodel.com/download/causalmediation.pdf
- Tingley, D., Yamamoto, T., Hirose, K., Keele, L., & Imai, K. (2014). Package ‘mediation’ [computer software]. http://cran.r-project.org/web/packages/mediation/

Intro to SCM

- Unifies:

  Parametric & nonparametric

  SEM and potential outcomes

  Data, graphical analysis

Intro to SCM

- Alternative to path analysis
- SEM program not needed
- But not latent variable models

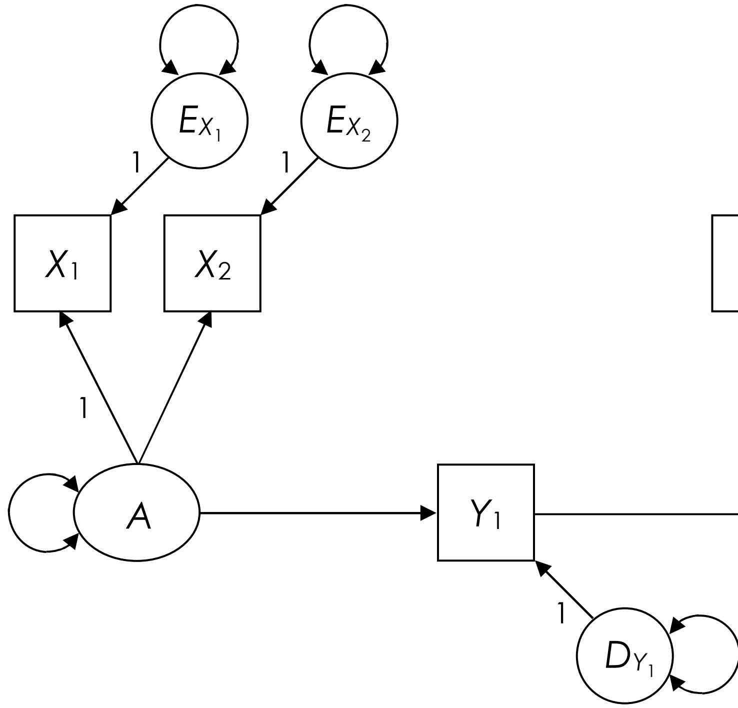
Intro to SCM

- Hayduk, L. A. & Littvay, L. (2012). Should researchers use single indicators, best indicators, or multiple indicators in structural equation models? *BMC Medical Research Methodology, 12*(159). http://www.biomedcentral.com/1471-2288/12/159

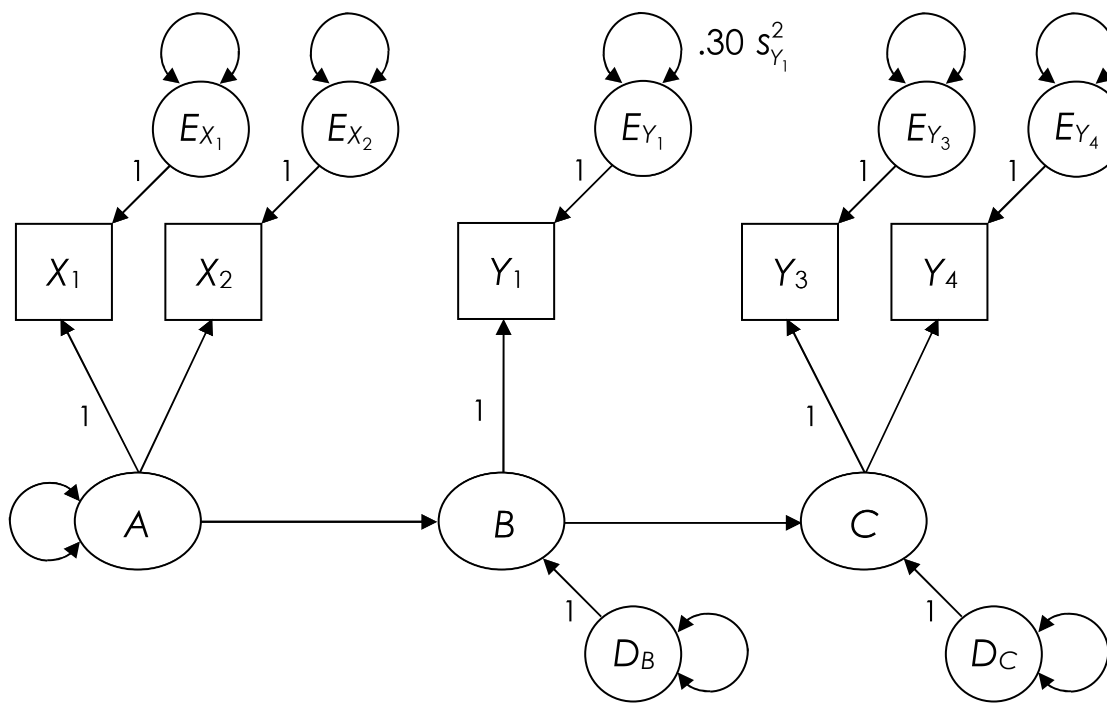
Intro to SCM

- Multiple indicators
- Some weak
- Best indicator is better

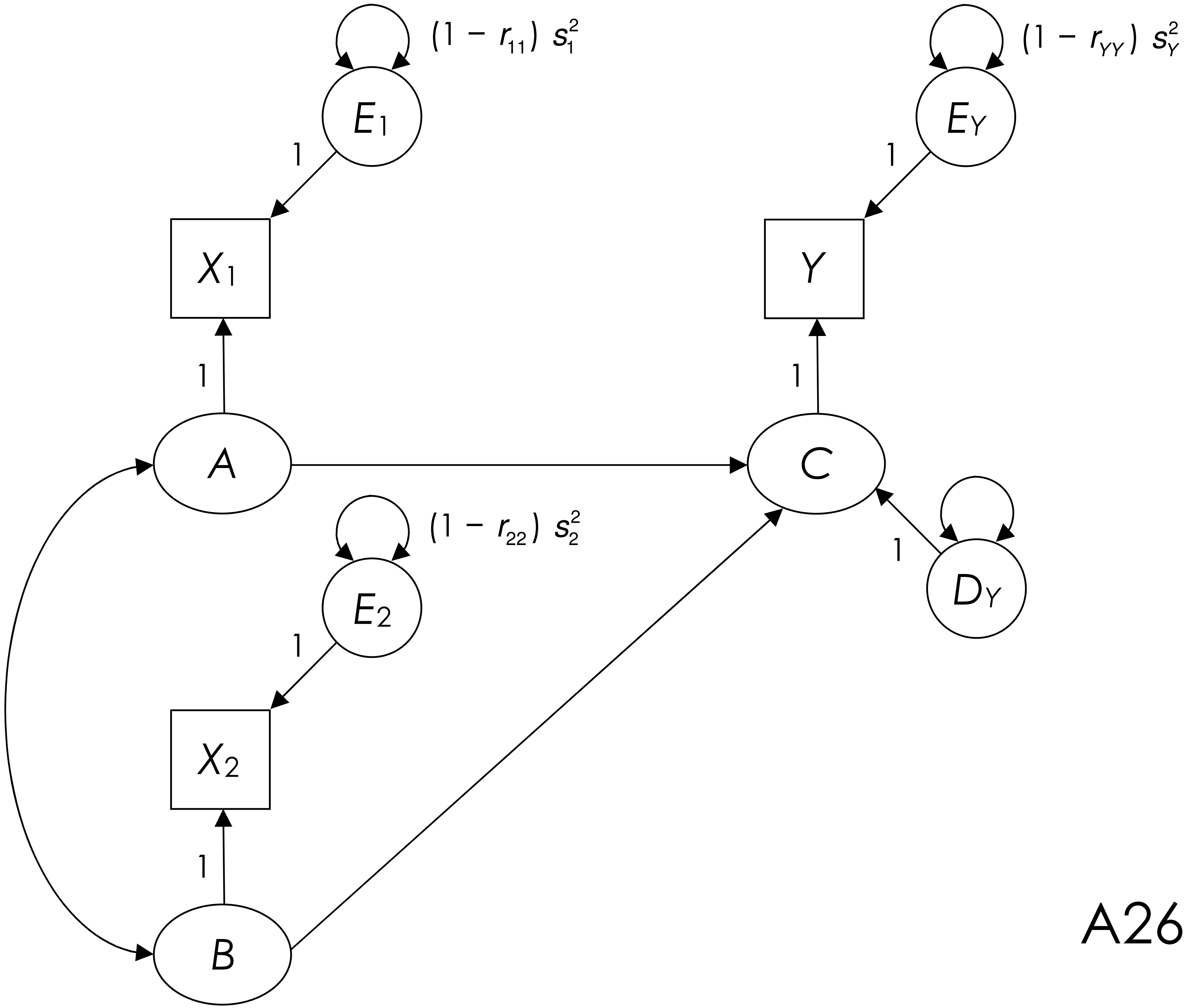
(a) Single endogenous indicator



(b) r_{YY} = .70 for Y_1



A25



A26

Intro to SCM

- Bayesian networks
- Graph structure
- Dependence relations

Intro to SCM

- Hypotheses as graphs
- Directly analyze graph (no data)
- Computer tools

Intro to SCM

- Nonparametric causal models
- Ideas (no operationalization)
- Study planning

Intro to SCM

- Directed acyclic graph (DAG)
- Directed cyclic graph (DCG)
- Recursive, nonrecursive

Intro to SCM

- Causal effects identified?
- If no, what should be measured?
- If yes, how many estimates?

Intro to SCM

- Identified model not required
- Estimate what you can
- Acknowledge info. gap for rest

Intro to SCM

- Regression analysis
- Causal model
- Covariate selection

Intro to SCM

- Bring on the data
- Model predictions
- Conditional independences

Intro to SCM

- All testable hypotheses
- Vanishing partial correlations
- Vanishing tetrads

SCM vocabulary

- Nodes, vertices (variables)
- Arcs, edges, links (paths)
- Adjacent (→), nonadjacent

SCM vocabulary

- Parents, ancestors
- Children, descendants
- Path is any sequence of edges

SCM vocabulary

- Directed path (causal)
- Undirected path (noncausal)

SCM vocabulary

- Open (unblocked) path
- Closed (blocked) path

SCM vocabulary

- Front-door path (causal)
- Back-door path (biasing)

SCM vocabulary

- Estimate causal effect
- Block all open biasing paths
- Do not open any blocked path

SCM vocabulary

- Ways to block or open paths:

    Covariates

    Sampling

Basic graphs

- Chain:

$$X \rightarrow W \rightarrow Y$$

Basic graphs

- Conditional independence:

$$X \rightarrow W \rightarrow Y$$

$$X \perp Y \mid W$$

Basic graphs

- Fork:

$$X \leftarrow W \rightarrow Y$$

Basic graphs

- Conditional independence:

$$X \leftarrow W \rightarrow Y$$

$$X \perp Y \mid W$$

Basic graphs

- Inverted fork (collider):

$$X \rightarrow W \leftarrow Y$$

$$X \perp Y$$

Basic graphs

- Special role of colliders:

  Controlling for a collider (or descendant) opens a blocked path

Basic graphs

- Special role of colliders:

  Controlling for a common outcome induces a spurious association between unrelated causes

Basic graphs

- Special role of colliders:

  Controlling for a common outcome adds a spurious component to related causes

Basic graphs

- Inverted fork (collider):

$$X \rightarrow W \leftarrow Y$$

$$X \perp Y$$

$$X \not\perp Y \mid W$$

Basic graphs

- Control for a collider (statistical):

$r_{XY} = 0$ $\quad$ $r_{YW} = .40$ $\quad$ $r_{XW} = .30$

$$r_{XY \cdot W} = -.14$$

Basic graphs

- Control for a collider (sampling):

  Speed $\rightarrow$ Fatalities $\leftarrow$ Alcohol

Basic graphs

- Descendant of a collider:

$$\begin{array}{ccccc} X & \rightarrow & W & \leftarrow & Y \\ & & \downarrow & & \\ & & A & & \end{array}$$

$$X \not\perp Y \mid A$$

Covariates

- Achen, C. H. (2005). Let's put garbage-can regressions and garbage-can probits where they belong. *Conflict Management and Peace Science*, *22*, 327–339.

Covariates

- Regression assumes:

  No causal effects
  between predictors

  Single equation

Basic graphs

- Overcontrol bias:

$$X \rightarrow W \rightarrow Y$$

$$X \perp Y \mid W$$

$$Y \text{ on } (X, W), \; B_X = 0$$

Basic graphs

- Endogenous selection bias:

$$X \rightarrow W \leftarrow Y$$

$$X \perp Y$$

$$Y \text{ on } (X, W), \;\; B_X \neq 0$$

d-Separation

- Conditional independences
- Testable implications
- Basis for identification

d-Separation

- Z d-separates X, Y if

  1. Z closes all open paths

  2. Z opens no blocked path

$$\underline{X} \rightarrow A \rightarrow \underline{B} \rightarrow Y$$

$$X \perp B \mid A$$

$$X \rightarrow \underline{A} \rightarrow \mathrm{B} \rightarrow \underline{Y}$$

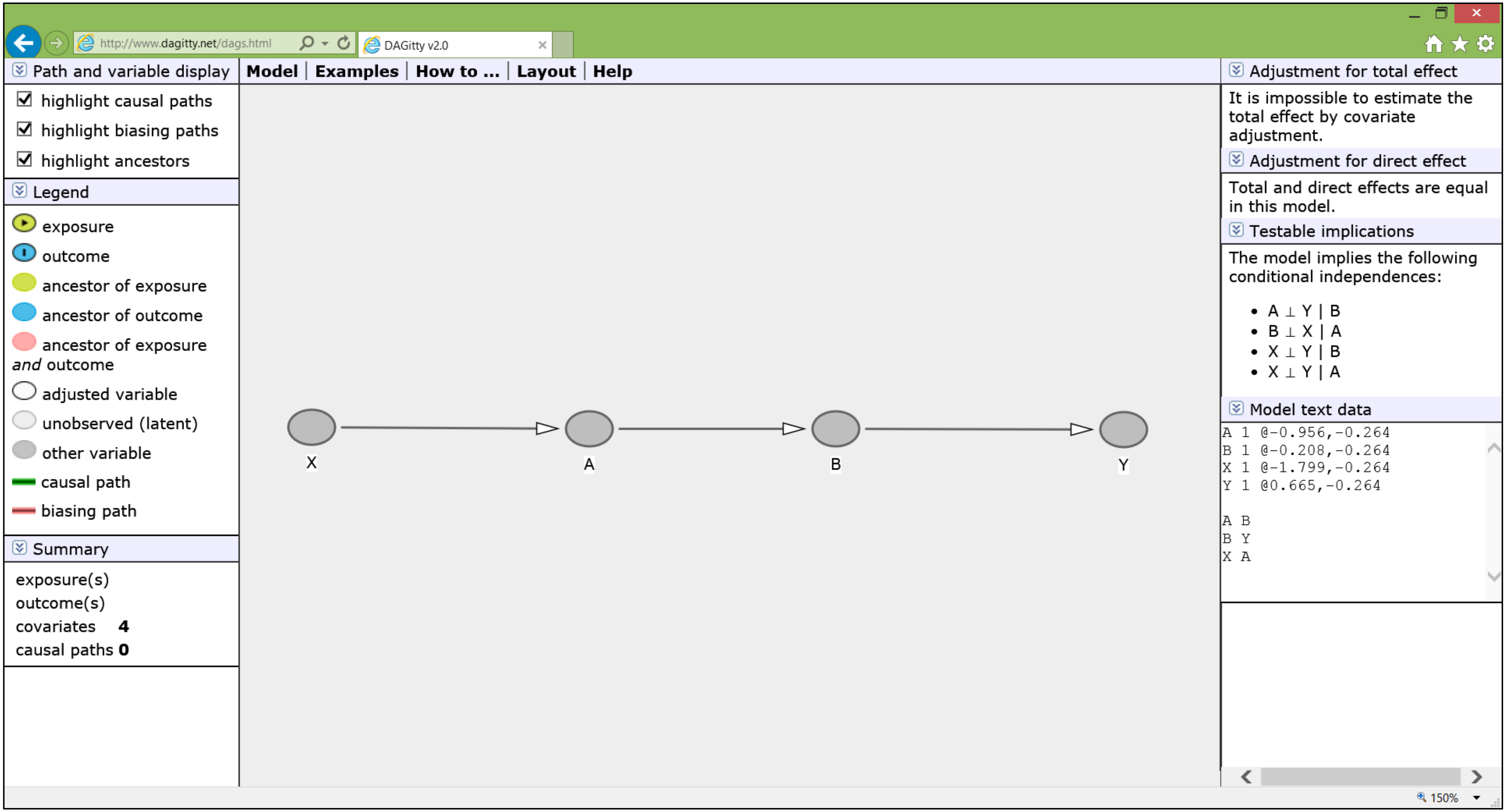
$$A \perp Y \mid B$$

$$\underline{X} \to A \to B \to \underline{Y}$$

$$X \perp Y \mid A$$

$$X \perp Y \mid B$$

$$X \perp Y \mid (A, B)$$


http://www.dagitty.net/dags.html
DAGitty v2.0
Path and variable display
Model | Examples | How to ... | Layout | Help
highlight causal paths
highlight biasing paths
highlight ancestors
Legend
exposure
outcome
ancestor of exposure
ancestor of outcome
ancestor of exposure and outcome
adjusted variable
unobserved (latent)
other variable
causal path
biasing path
Summary
exposure(s)
outcome(s)
covariates 4
causal paths 0
X
A
B
Y
Adjustment for total effect
It is impossible to estimate the total effect by covariate adjustment.
Adjustment for direct effect
Total and direct effects are equal in this model.
Testable implications
The model implies the following conditional independences:
A ⊥ Y | B
B ⊥ X | A
X ⊥ Y | B
X ⊥ Y | A
Model text data
A 1 @-0.956,-0.264
B 1 @-0.208,-0.264
X 1 @-1.799,-0.264
Y 1 @0.665,-0.264
A B
B Y
X A
150%

A64

$$\underline{X} \longrightarrow A \longleftarrow \underline{B} \longrightarrow Y$$

$$X \perp B$$

$$X \not\perp B \mid A$$

$$X \rightarrow \underline{A} \leftarrow B \rightarrow \underline{Y}$$

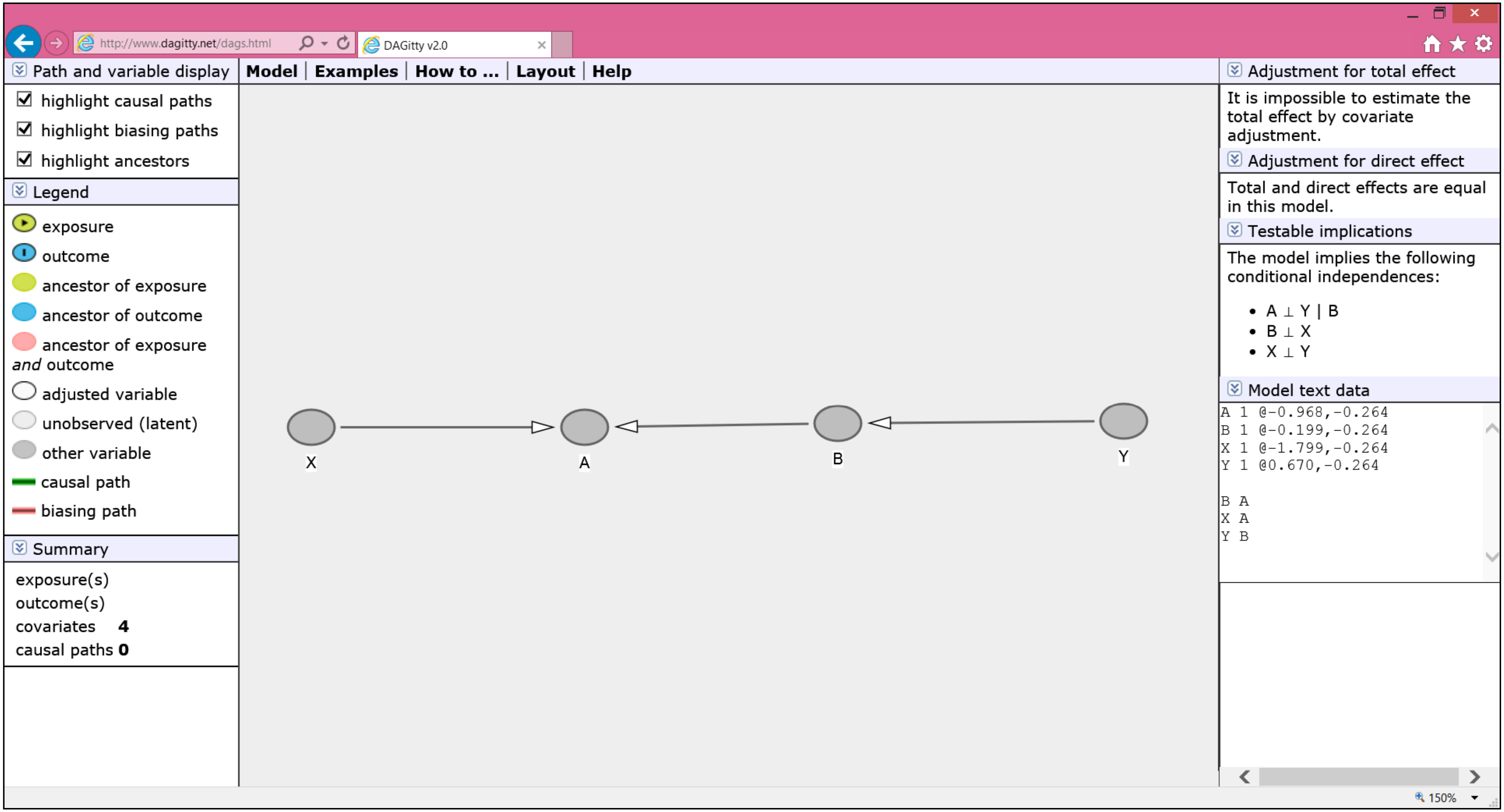
$$A \perp Y \mid B$$

$$\underline{X} \rightarrow A \leftarrow B \rightarrow \underline{Y}$$

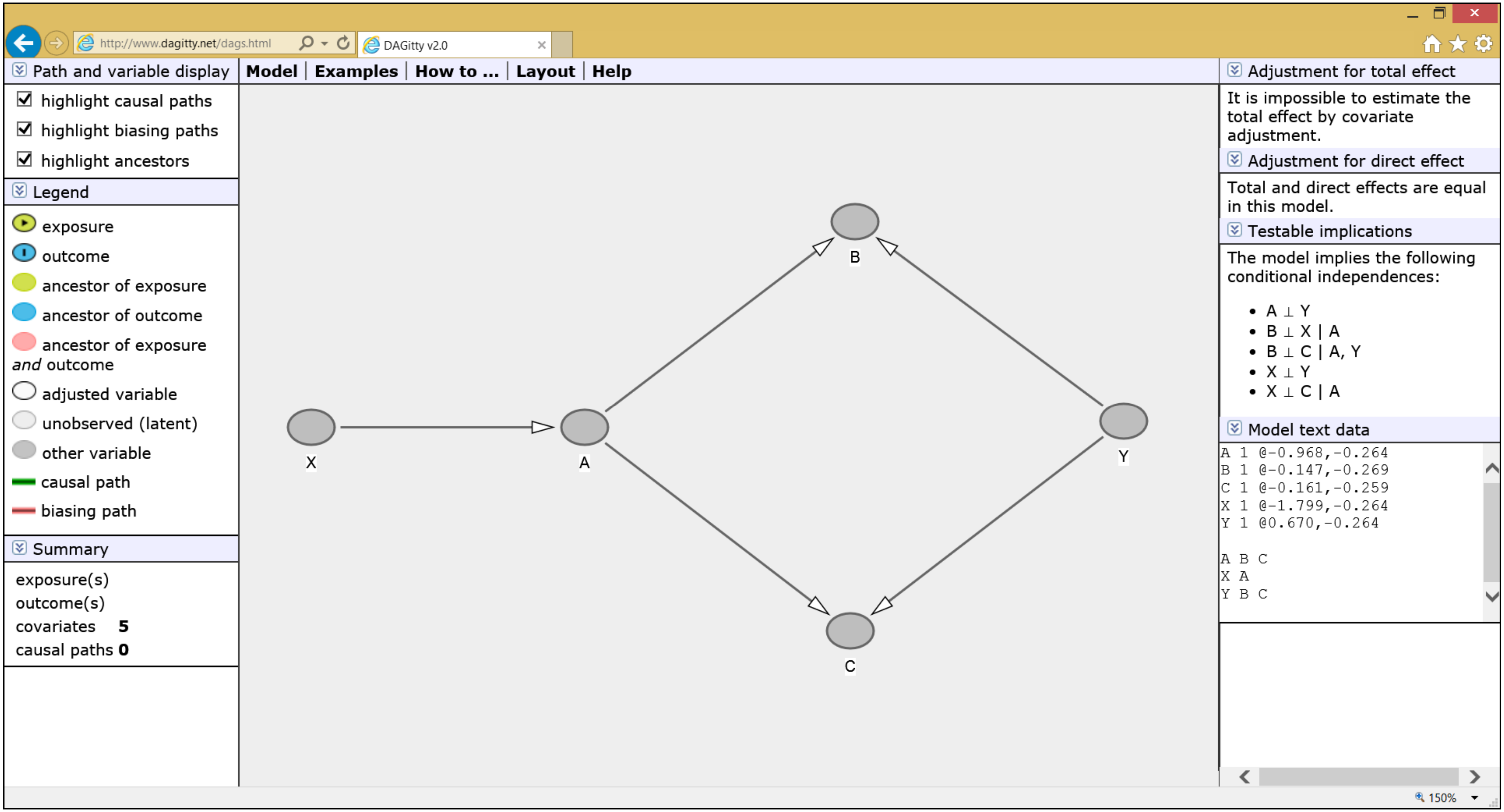
$$X \perp Y$$

$$X \perp Y \mid B$$

$$X \not\perp Y \mid A$$


http://www.dagitty.net/dags.html
DAGitty v2.0
Path and variable display
Model | Examples | How to ... | Layout | Help
highlight causal paths
highlight biasing paths
highlight ancestors
Legend
exposure
outcome
ancestor of exposure
ancestor of outcome
ancestor of exposure and outcome
adjusted variable
unobserved (latent)
other variable
causal path
biasing path
Summary
exposure(s)
outcome(s)
covariates 4
causal paths 0
X
A
B
Y
Adjustment for total effect
It is impossible to estimate the total effect by covariate adjustment.
Adjustment for direct effect
Total and direct effects are equal in this model.
Testable implications
The model implies the following conditional independences:
A ⊥ Y | B
B ⊥ X
X ⊥ Y
Model text data
A 1 @-0.968,-0.264
B 1 @-0.199,-0.264
X 1 @-1.799,-0.264
Y 1 @0.670,-0.264
B A
X A
Y B
150%

A68


http://www.dagitty.net/dags.html
DAGitty v2.0
Path and variable display
Model | Examples | How to ... | Layout | Help
highlight causal paths
highlight biasing paths
highlight ancestors
Legend
exposure
outcome
ancestor of exposure
ancestor of outcome
ancestor of exposure and outcome
adjusted variable
unobserved (latent)
other variable
causal path
biasing path
Summary
exposure(s)
outcome(s)
covariates 5
causal paths 0
B
X
A
Y
C
Adjustment for total effect
It is impossible to estimate the total effect by covariate adjustment.
Adjustment for direct effect
Total and direct effects are equal in this model.
Testable implications
The model implies the following conditional independences:
A ⊥ Y
B ⊥ X | A
B ⊥ C | A, Y
X ⊥ Y
X ⊥ C | A
Model text data
A 1 @-0.968,-0.264
B 1 @-0.147,-0.269
C 1 @-0.161,-0.259
X 1 @-1.799,-0.264
Y 1 @0.670,-0.264
A B C
X A
Y B C
150%

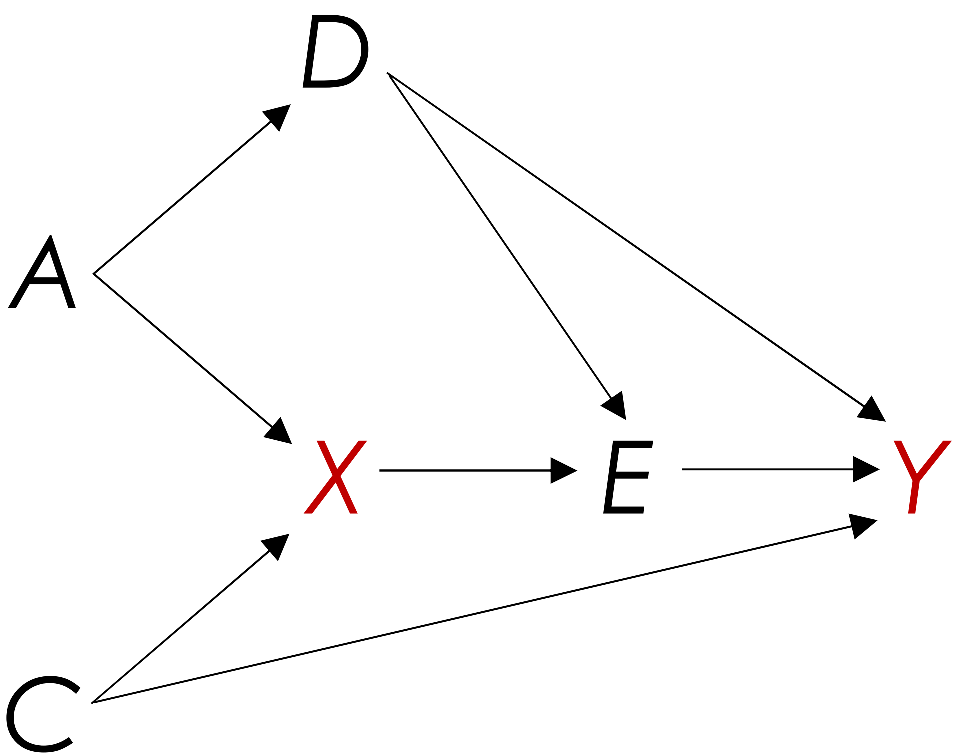
A69

Identification

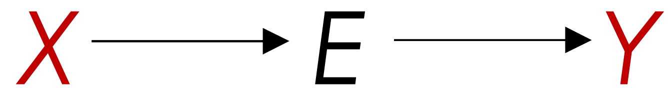
- Graphical criteria
- Sufficient (deconfounding) set
- Removes all noncausal aspects

Identification

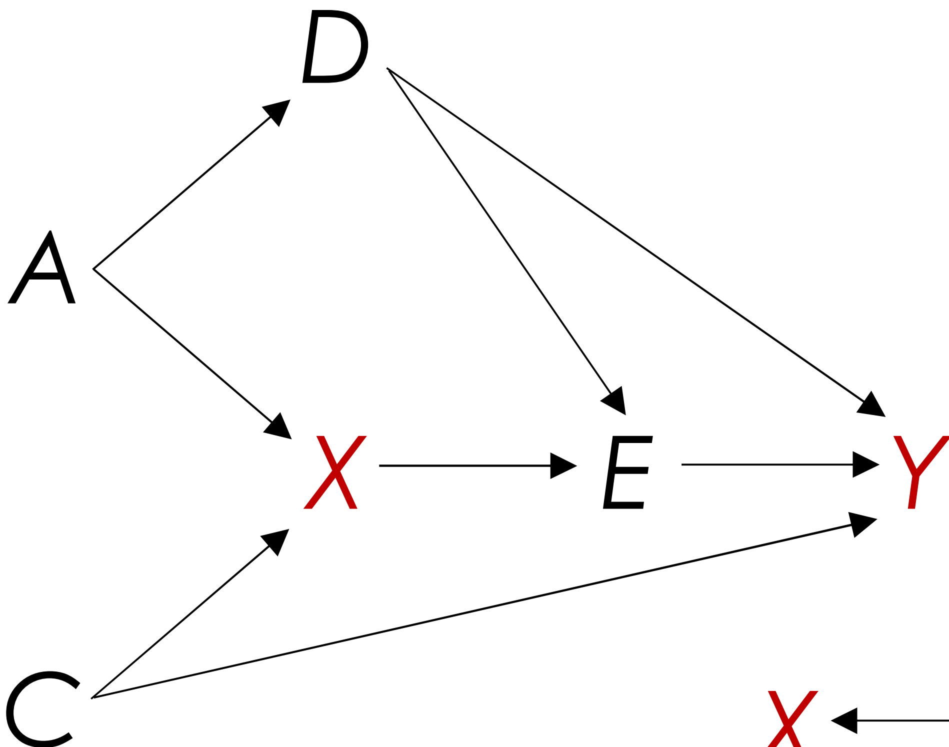
- Back-door criterion (total effects)
- Closes biasing (back-door) paths
- Leaves only causal



Causal



A72

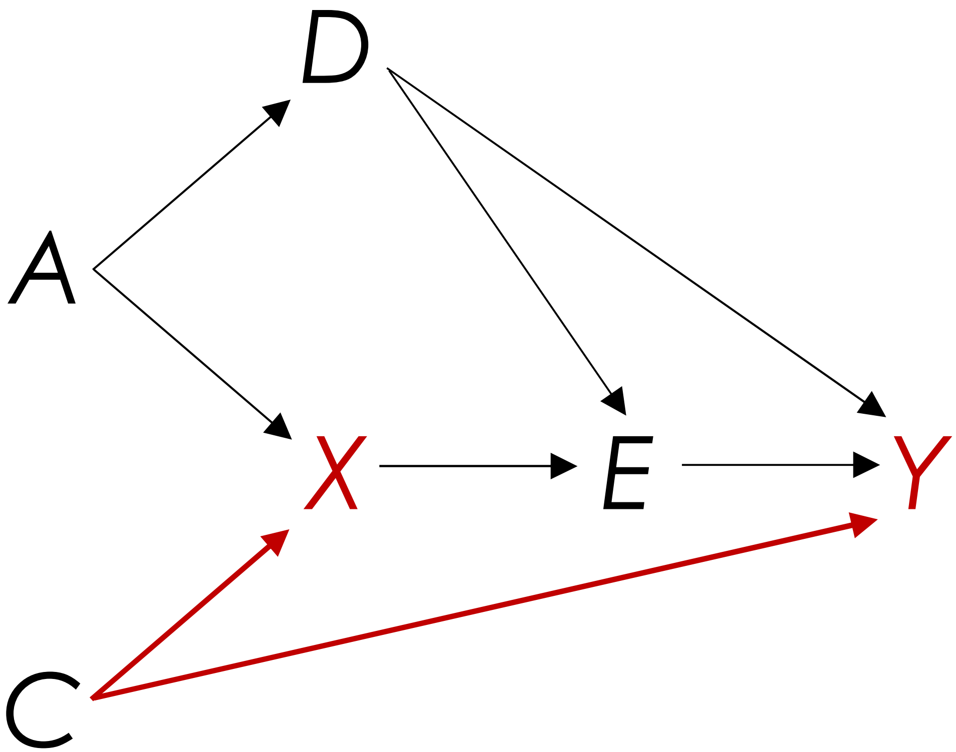


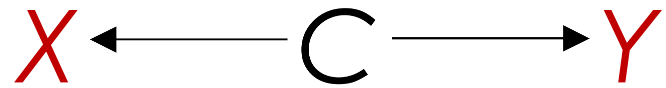
Back-door

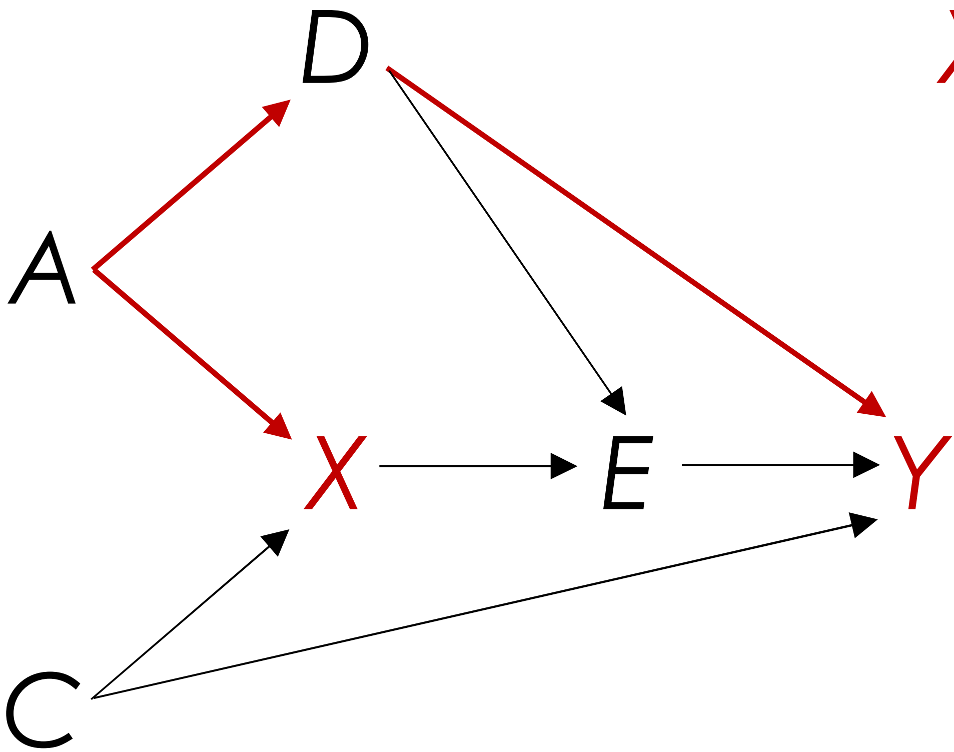
$$X \leftarrow C \rightarrow Y$$

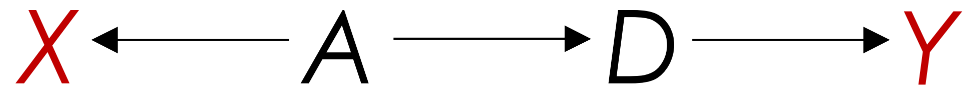
$$X \leftarrow A \rightarrow D \rightarrow Y$$

$$X \leftarrow A \rightarrow D \rightarrow E \rightarrow Y$$

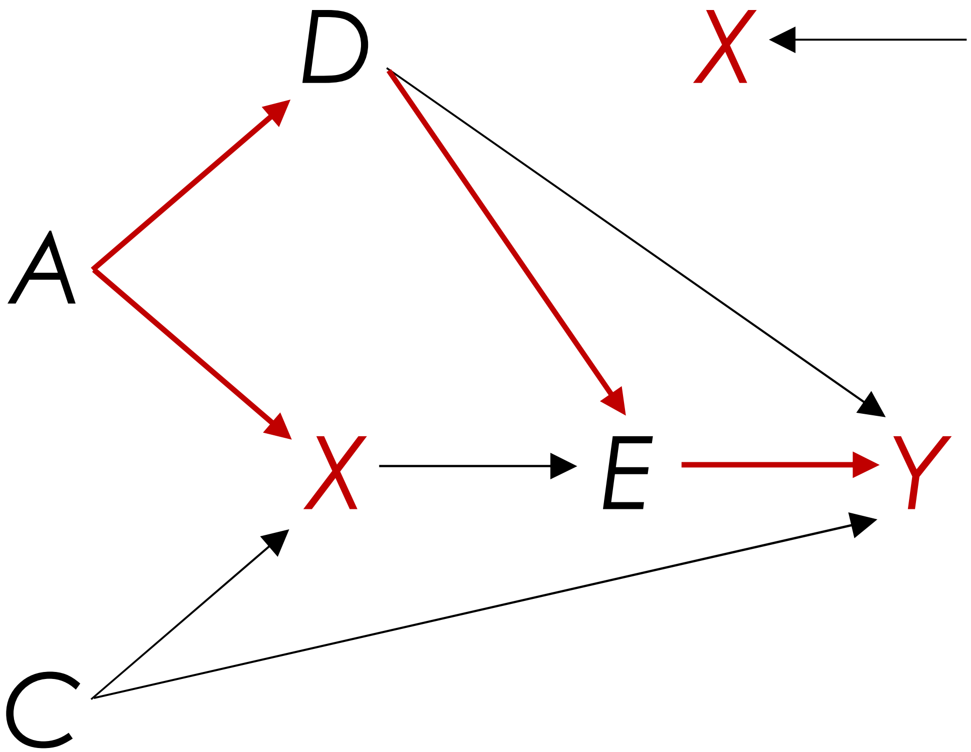

D
A
X
E
Y
C

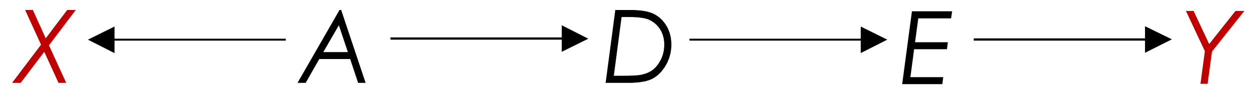

X
C
Y





A75


D
A
X
E
Y
C


X
A
D
E
Y

Back-door

$$X \leftarrow C \rightarrow Y$$

$$X \leftarrow A \rightarrow D \rightarrow Y$$

$$X \leftarrow A \rightarrow D \rightarrow E \rightarrow Y$$

Y on (X, A, C)

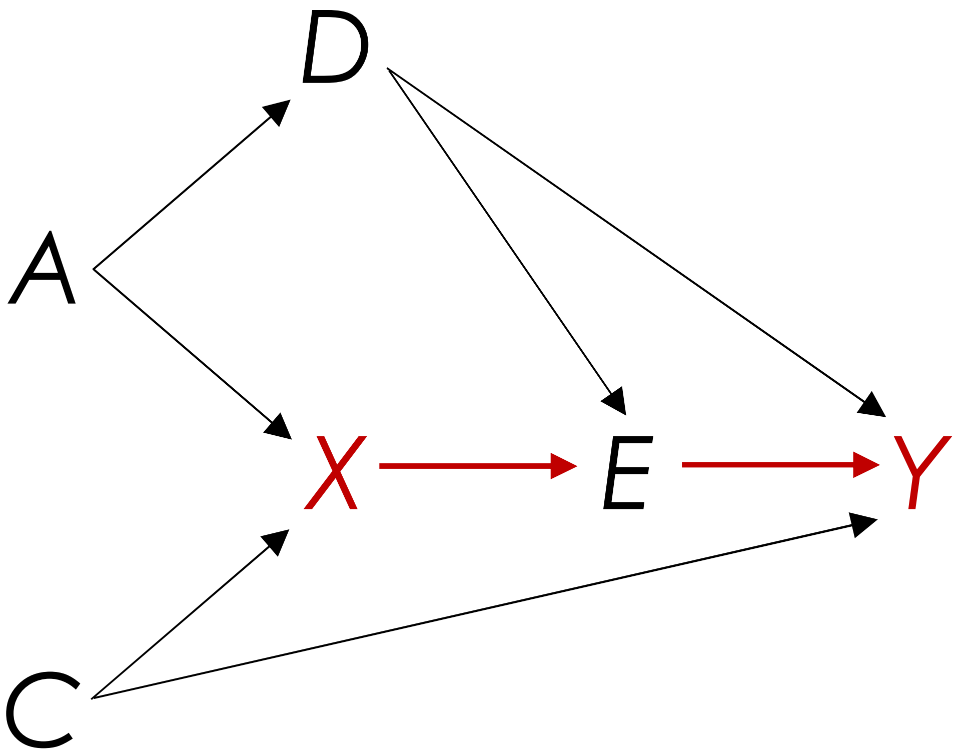
Back-door

$$X \leftarrow C \rightarrow Y$$

$$X \leftarrow A \rightarrow D \rightarrow Y$$

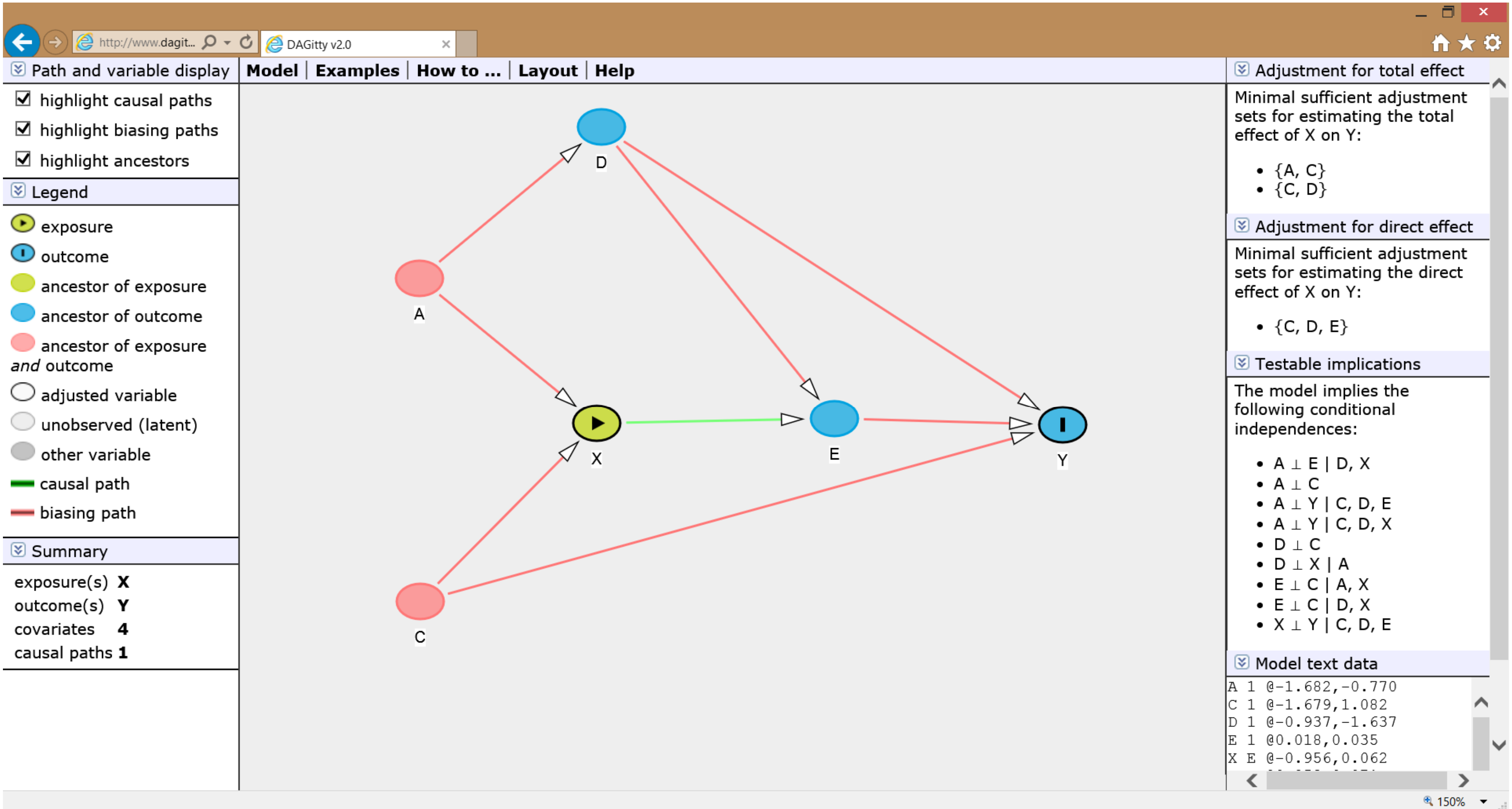
$$X \leftarrow A \rightarrow D \rightarrow E \rightarrow Y$$

Y on (X, C, D)



Y on (X, A, C)

Y on (X, C, D)


http://www.dagit...
DAGitty v2.0
Path and variable display
Model
Examples
How to ...
Layout
Help
highlight causal paths
highlight biasing paths
highlight ancestors
Legend
exposure
outcome
ancestor of exposure
ancestor of outcome
ancestor of exposure *and* outcome
adjusted variable
unobserved (latent)
other variable
causal path
biasing path
Summary
exposure(s) X
outcome(s) Y
covariates 4
causal paths 1
D
A
X
E
Y
C
Adjustment for total effect
Minimal sufficient adjustment sets for estimating the total effect of X on Y:
{A, C}
{C, D}
Adjustment for direct effect
Minimal sufficient adjustment sets for estimating the direct effect of X on Y:
{C, D, E}
Testable implications
The model implies the following conditional independences:
A ⊥ E | D, X
A ⊥ C
A ⊥ Y | C, D, E
A ⊥ Y | C, D, X
D ⊥ C
D ⊥ X | A
E ⊥ C | A, X
E ⊥ C | D, X
X ⊥ Y | C, D, E
Model text data
A 1 @-1.682,-0.770
C 1 @-1.679,1.082
D 1 @-0.937,-1.637
E 1 @0.018,0.035
X E @-0.956,0.062
150%

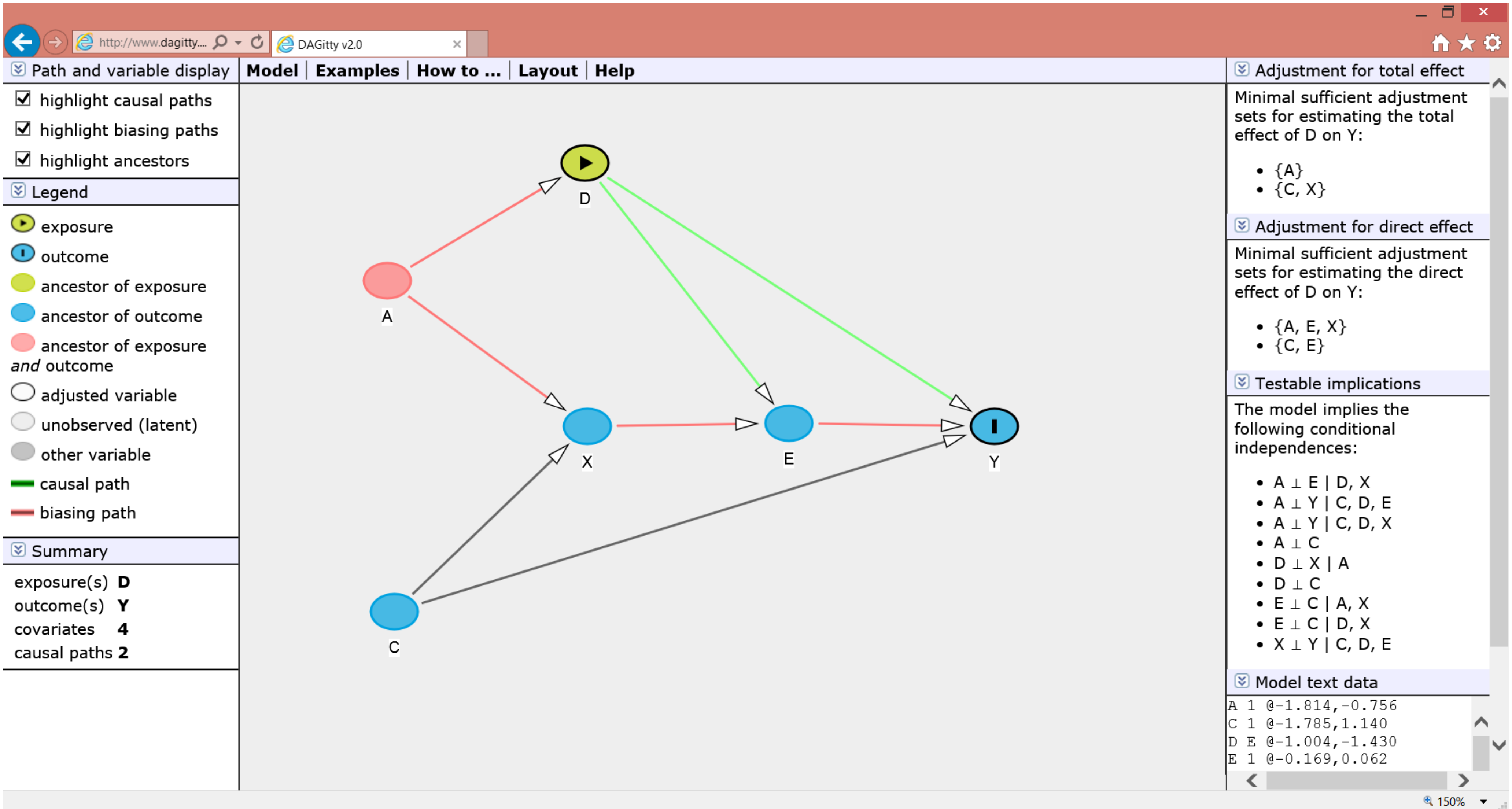
A80

Identification

- Single-door criterion (direct)
- Recursive, linear, continuous
- Multiple estimates

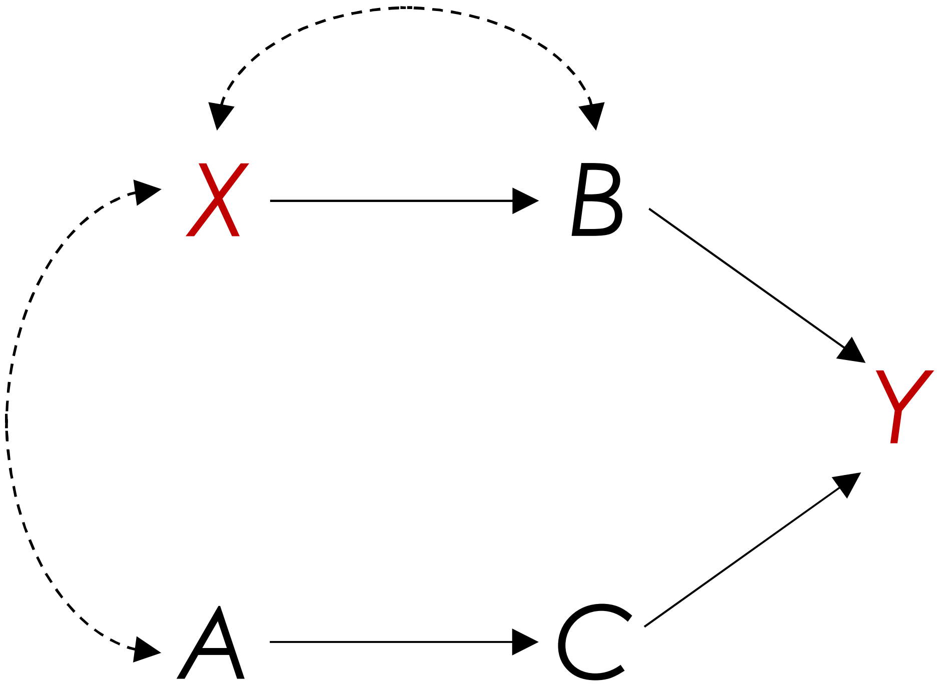
Identification

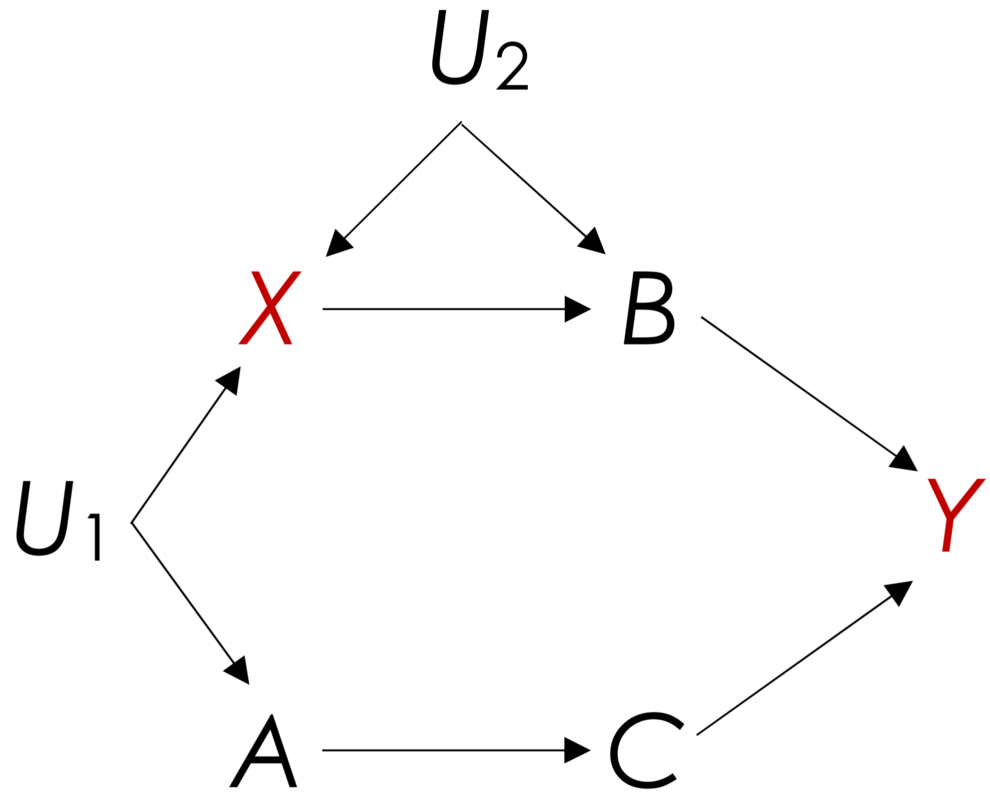
- Single-door criterion ($D \to Y$)
- Delete $\to$
- Sufficient set d-separates



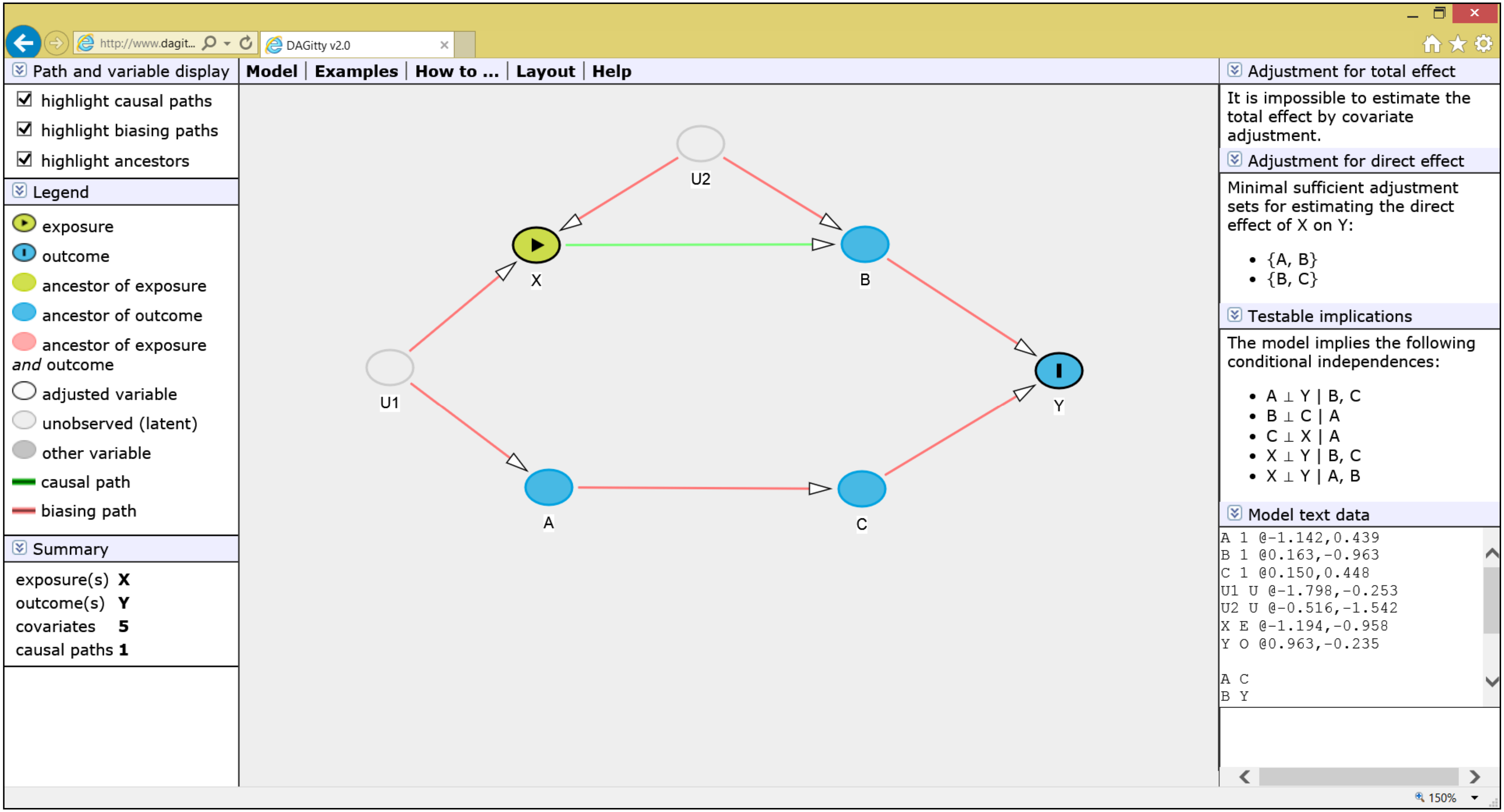
http://www.dagitty....
DAGitty v2.0
Path and variable display
highlight causal paths
highlight biasing paths
highlight ancestors
Legend
exposure
outcome
ancestor of exposure
ancestor of outcome
ancestor of exposure *and* outcome
adjusted variable
unobserved (latent)
other variable
causal path
biasing path
Summary
exposure(s) D
outcome(s) Y
covariates 4
causal paths 2
Model | Examples | How to ... | Layout | Help
D
A
X
E
Y
C
Adjustment for total effect
Minimal sufficient adjustment sets for estimating the total effect of D on Y:
{A}
{C, X}
Adjustment for direct effect
Minimal sufficient adjustment sets for estimating the direct effect of D on Y:
{A, E, X}
{C, E}
Testable implications
The model implies the following conditional independences:
A ⊥ E | D, X
A ⊥ Y | C, D, E
A ⊥ Y | C, D, X
A ⊥ C
D ⊥ X | A
D ⊥ C
E ⊥ C | A, X
E ⊥ C | D, X
X ⊥ Y | C, D, E
Model text data
A 1 @-1.814,-0.756
C 1 @-1.785,1.140
D E @-1.004,-1.430
E 1 @-0.169,0.062
150%

A83

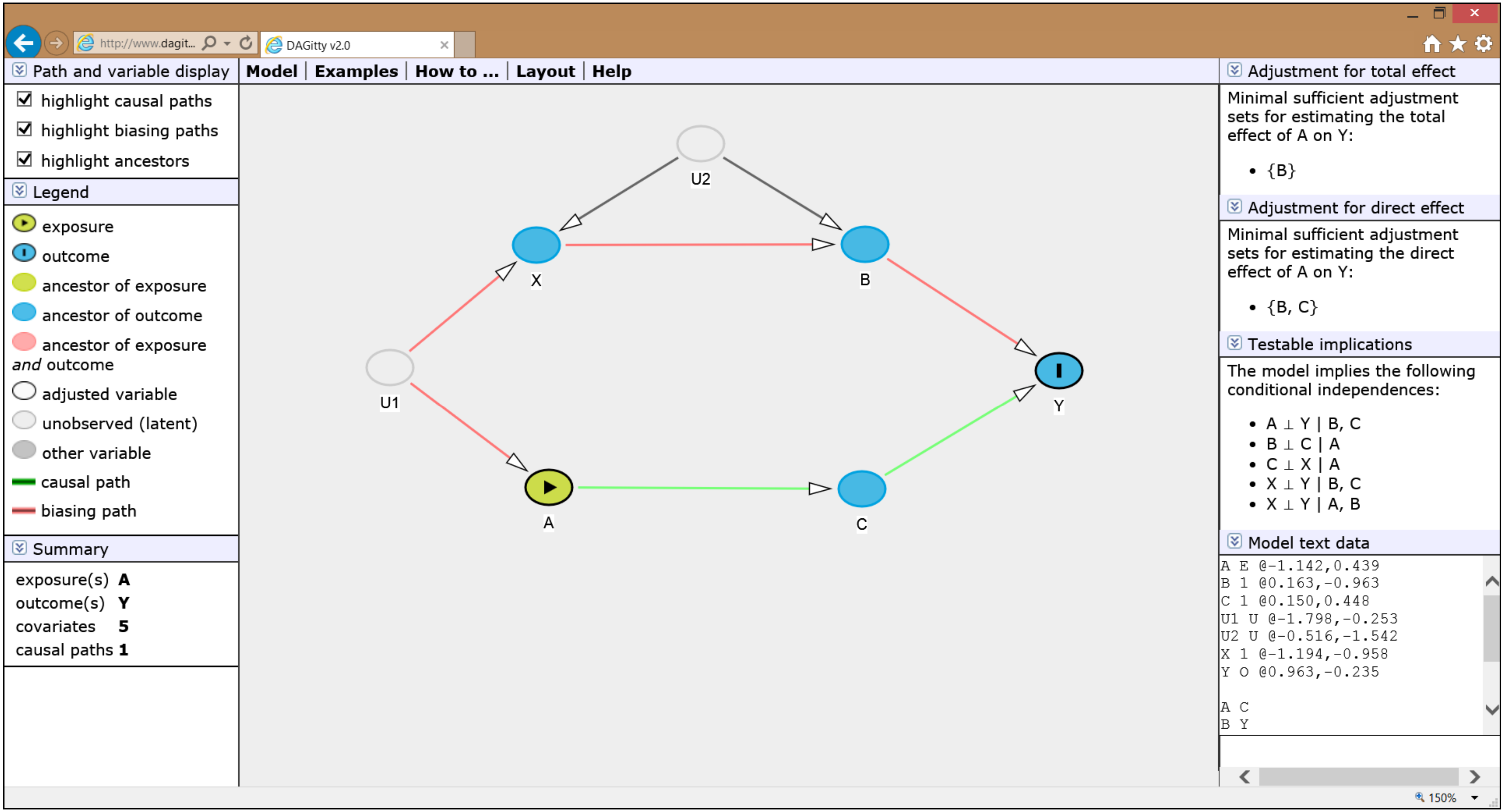




A84


DAGitty v2.0
Path and variable display
highlight causal paths
highlight biasing paths
highlight ancestors
Legend
exposure
outcome
ancestor of exposure
ancestor of outcome
ancestor of exposure and outcome
adjusted variable
unobserved (latent)
other variable
causal path
biasing path
Summary
exposure(s) X
outcome(s) Y
covariates 5
causal paths 1
Model | Examples | How to ... | Layout | Help
U2
X
B
U1
Y
A
C
Adjustment for total effect
It is impossible to estimate the total effect by covariate adjustment.
Adjustment for direct effect
Minimal sufficient adjustment sets for estimating the direct effect of X on Y:
{A, B}
{B, C}
Testable implications
The model implies the following conditional independences:
A ⊥ Y | B, C
B ⊥ C | A
C ⊥ X | A
X ⊥ Y | B, C
X ⊥ Y | A, B
Model text data
A 1 @-1.142,0.439
B 1 @0.163,-0.963
C 1 @0.150,0.448
U1 U @-1.798,-0.253
U2 U @-0.516,-1.542
X E @-1.194,-0.958
Y O @0.963,-0.235
A C
B Y
150%

A85



http://www.dagit...
DAGitty v2.0
Path and variable display
Model | Examples | How to ... | Layout | Help
Adjustment for total effect
highlight causal paths
highlight biasing paths
highlight ancestors
Legend
exposure
outcome
ancestor of exposure
ancestor of outcome
ancestor of exposure and outcome
adjusted variable
unobserved (latent)
other variable
causal path
biasing path
Summary
exposure(s) A
outcome(s) Y
covariates 5
causal paths 1
U2
X
B
U1
Y
A
C
Minimal sufficient adjustment sets for estimating the total effect of A on Y:
• {B}
Adjustment for direct effect
Minimal sufficient adjustment sets for estimating the direct effect of A on Y:
• {B, C}
Testable implications
The model implies the following conditional independences:
• A ⊥ Y | B, C
• B ⊥ C | A
• C ⊥ X | A
• X ⊥ Y | B, C
• X ⊥ Y | A, B
Model text data
A E @-1.142,0.439
B 1 @0.163,-0.963
C 1 @0.150,0.448
U1 U @-1.798,-0.253
U2 U @-0.516,-1.542
X 1 @-1.194,-0.958
Y O @0.963,-0.235
A C
B Y
150%

A86

Identification

- Merchant, A. T, & Pitiphat, W. (2002). Directed acyclic graphs (DAGs): An aid to assess confounding in dental research. Community Dentistry and Oral Epidemiology, 30: 399–404.
- Fleischer, N. L., & Diez Roux , A. V. (2008). Using directed acyclic graphs to guide analyses of neighbourhood health effects: An introduction. *Journal of Epidemiology & Community Health*, 62, 842-846.

Analysis

- Roth, D. L., Wiebe, D. J., Fillingim, R. B., & Shay, K. A. (1989). Life events, fitness, hardiness, and health: A simultaneous analysis of proposed stress-resistance effects. *Journal of Personality and Social Psychology*, *57*, 136–142.

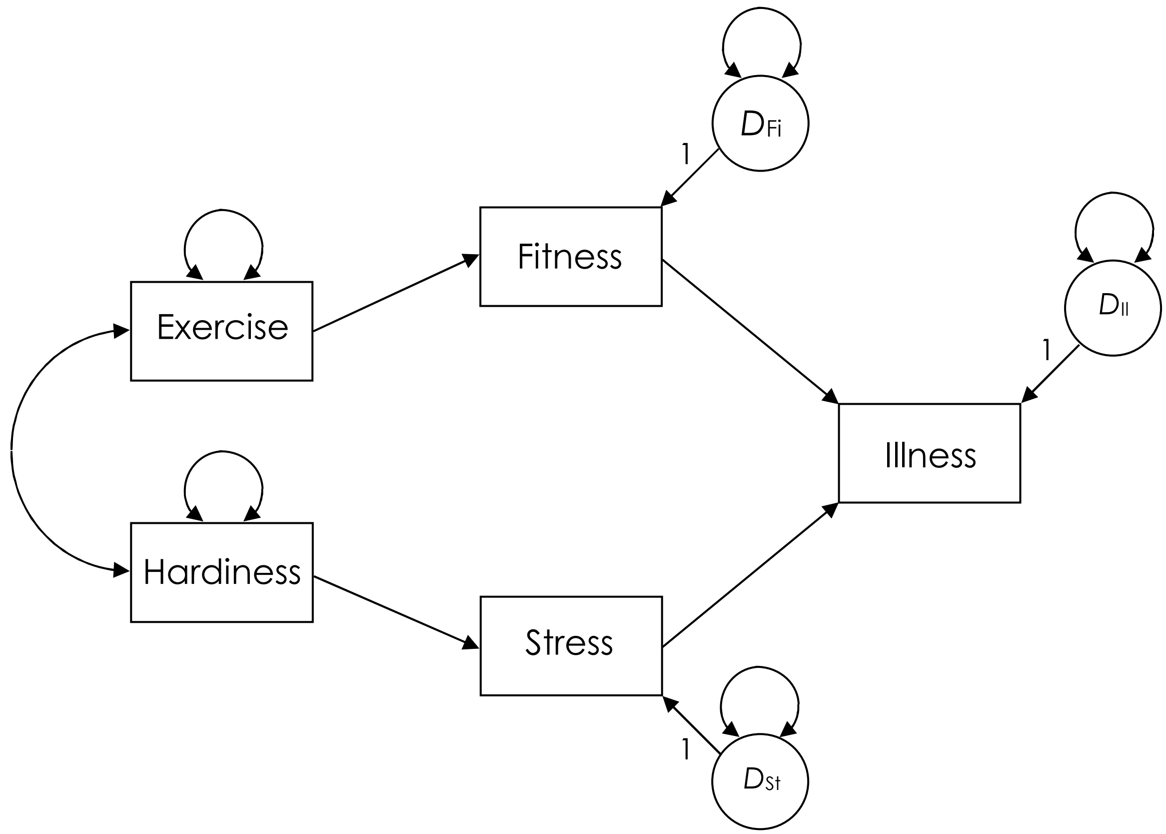


Figure 8.5. A recursive path model of health factors.

Variable	1	2	3	4	5
1. Exercise	—				
2. Hardiness	−.03	—			
3. Fitness	.39	.07	—		
4. Stress	−.05	−.23	−.13	—	
5. Illness	−.08	−.16	−.29	.34	—
M	40.90	0.0	67.10	4.80	716.70
SD	66.50	38.00	18.40	33.50	62.48

Note. These data (correlations, means, and variances) are from D. Roth et al. (1989); $N = 373$.

Independence	Conditioning set	Partial correlation
Exercise ⊥ Stress	Hardiness	−.058
Exercise ⊥ Illness	Fitness, Stress	.039
Hardiness ⊥ Fitness	Exercise	.089
Hardiness ⊥ Illness	Fitness, Stress	−.081
Fitness ⊥ Stress	Exercise, Hardiness	**−.103**

	Minimally sufficient set				
Direct effect	∅	Exercise	Hardiness	Stress	Fitness
Exercise → Fitness	.108 (.013) .390	—	—	—	—
Hardiness → Stress	−.203 (.045) −.230	—	—	—	—
Fitness → Illness	—	−1.036 (.183) −.305	−.951 (.168) −.280	**−.849 (.162) −.250**	—
Stress → Illness	—	.628 (.091) .337	.597 (.093) .320	—	**.574 (.089) .307**

Note. Estimates are reported as unstandardized (standard error) standardized; ∅, empty set. Values in boldface control for the all parents of each outcome.

Extensions

- Locate instruments
- Models → counterfactuals
- Potential outcomes (PO)

Strengths

- Unifying model (SEM, PO)
- Supports reasoning, planning
- Local fit, not global

Limitations

- Classical measurement
- No global fit
- Few software tools


Thank You