

New developments in structural equation modeling

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Set B: Mediation

UNL Methodology Workshop

Welcome
back!



Topics

- Mediation:

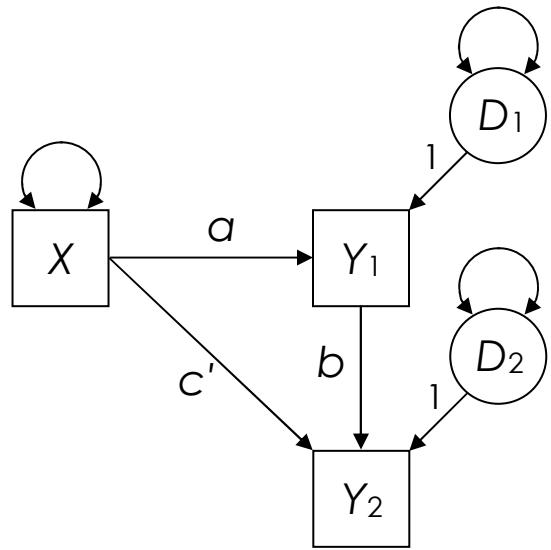
- Design requirements

- Conditional process modeling

- Cause × mediator

Design

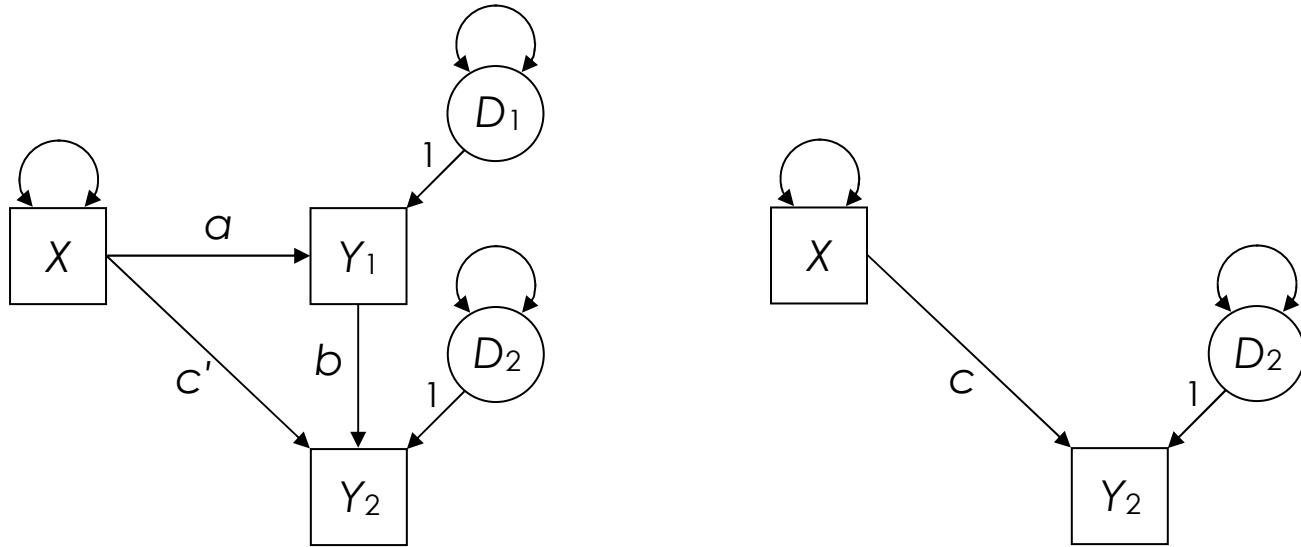
- Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic and statistical considerations. *Journal of Personality and Social Psychology, 51*, 1173–1182.



$$X \rightarrow Y_1 \rightarrow Y_2 = ab$$

c'

A5



$c, a \ * \ *$

$c' \text{ not } * \ (c' < c)$

$ab \ * \ *$

A6

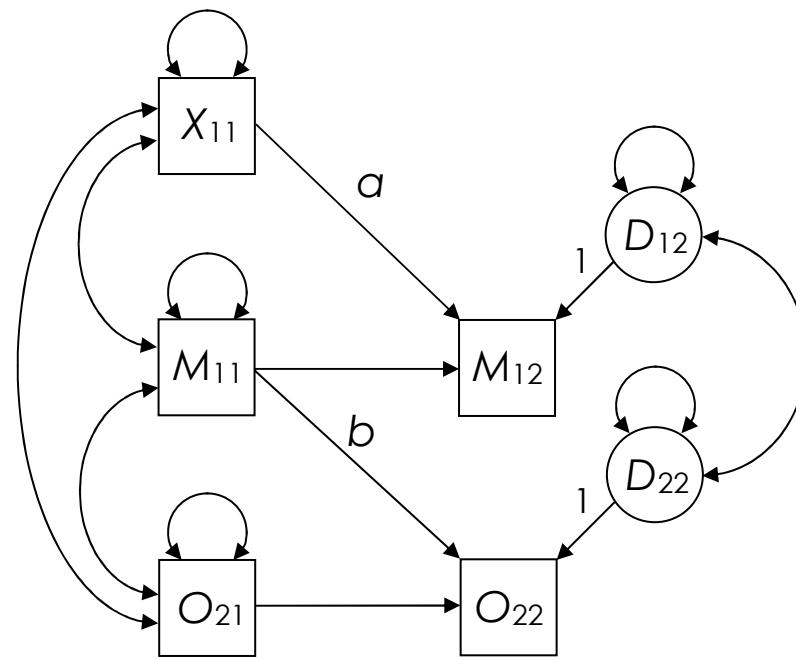
Design

- Newer view:
 - * insufficient
 - Time precedence
 - Changes

Mediation refers to the causal hypothesis that one variable causes **changes** in another variable, which in turn leads to **changes** in the outcome variable.

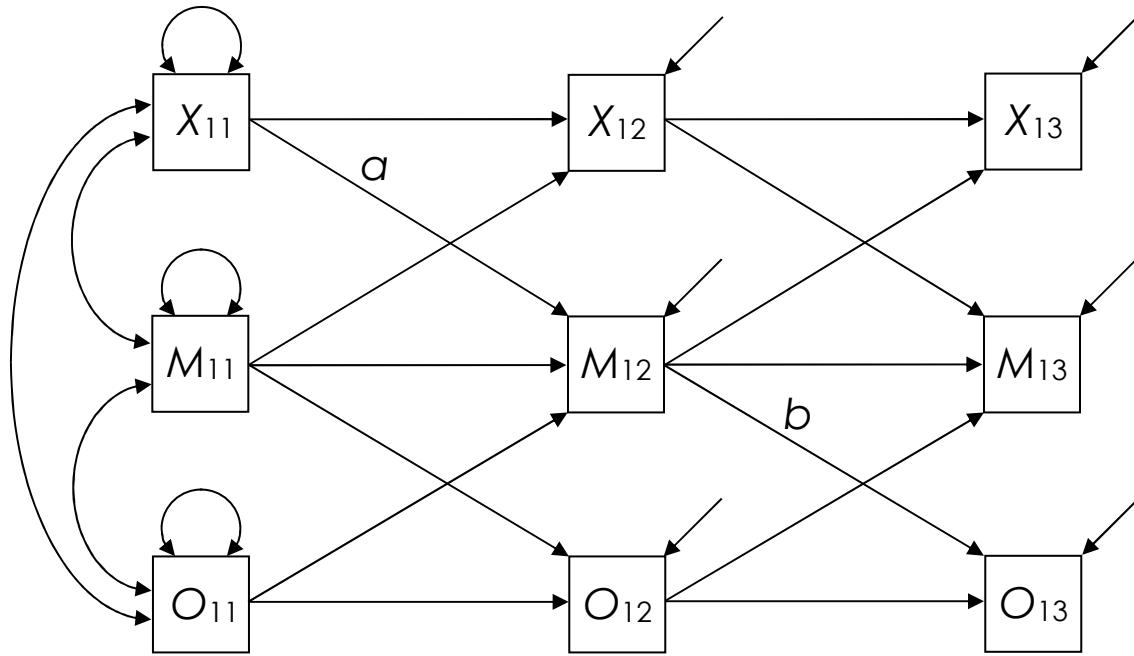
Little (2013)

Half longitudinal mediation



A9

Full longitudinal mediation

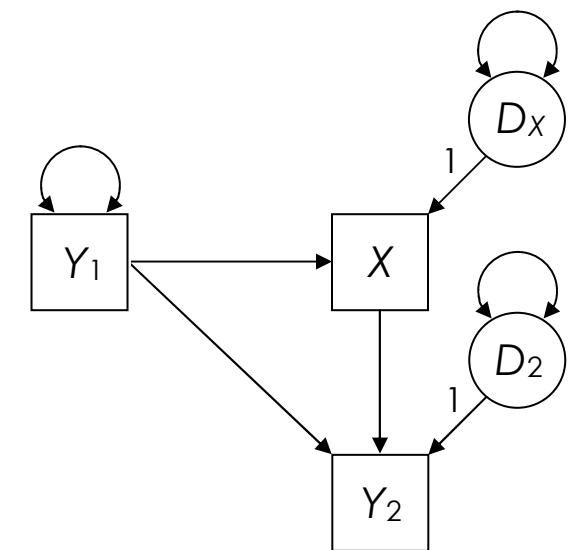
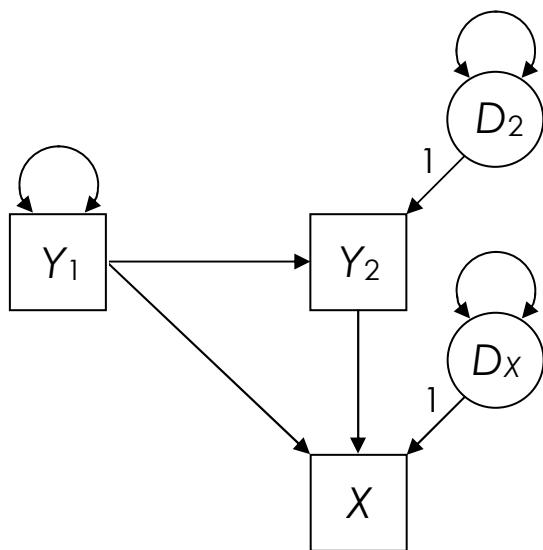
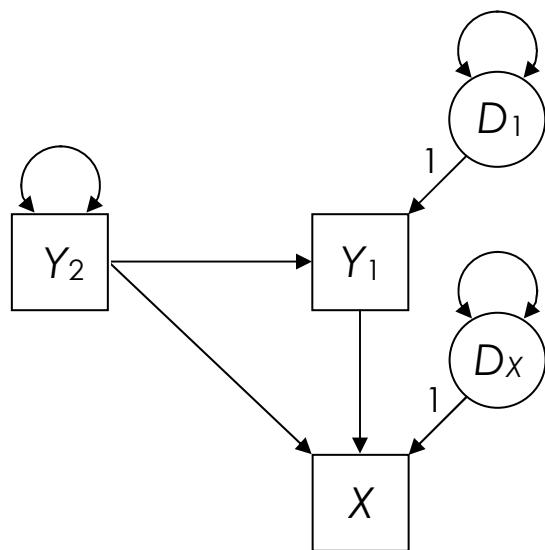
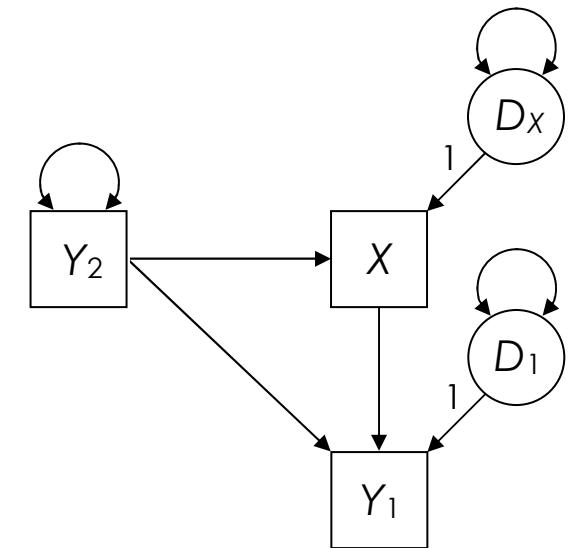
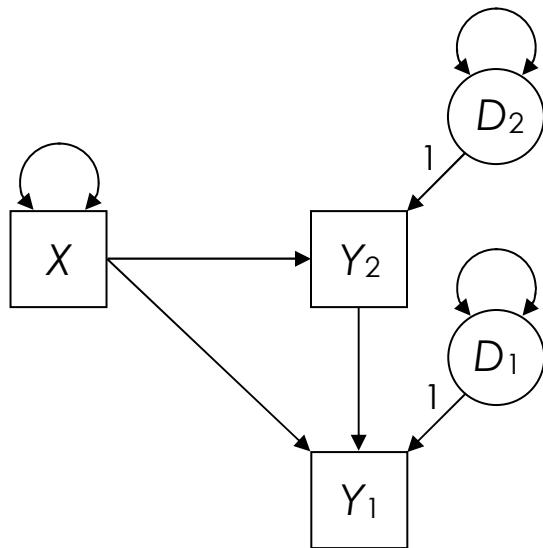
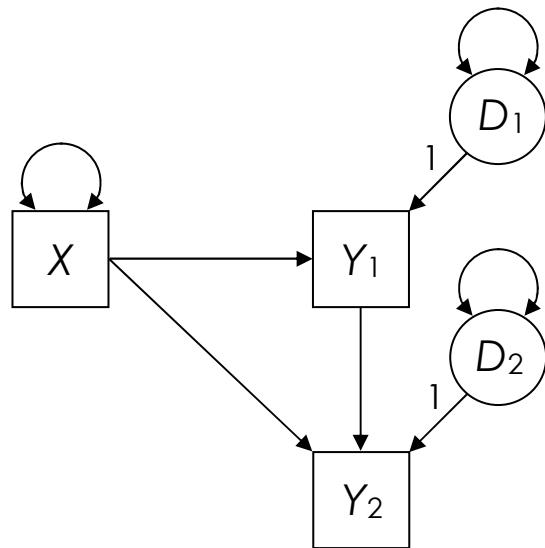


A10

Design

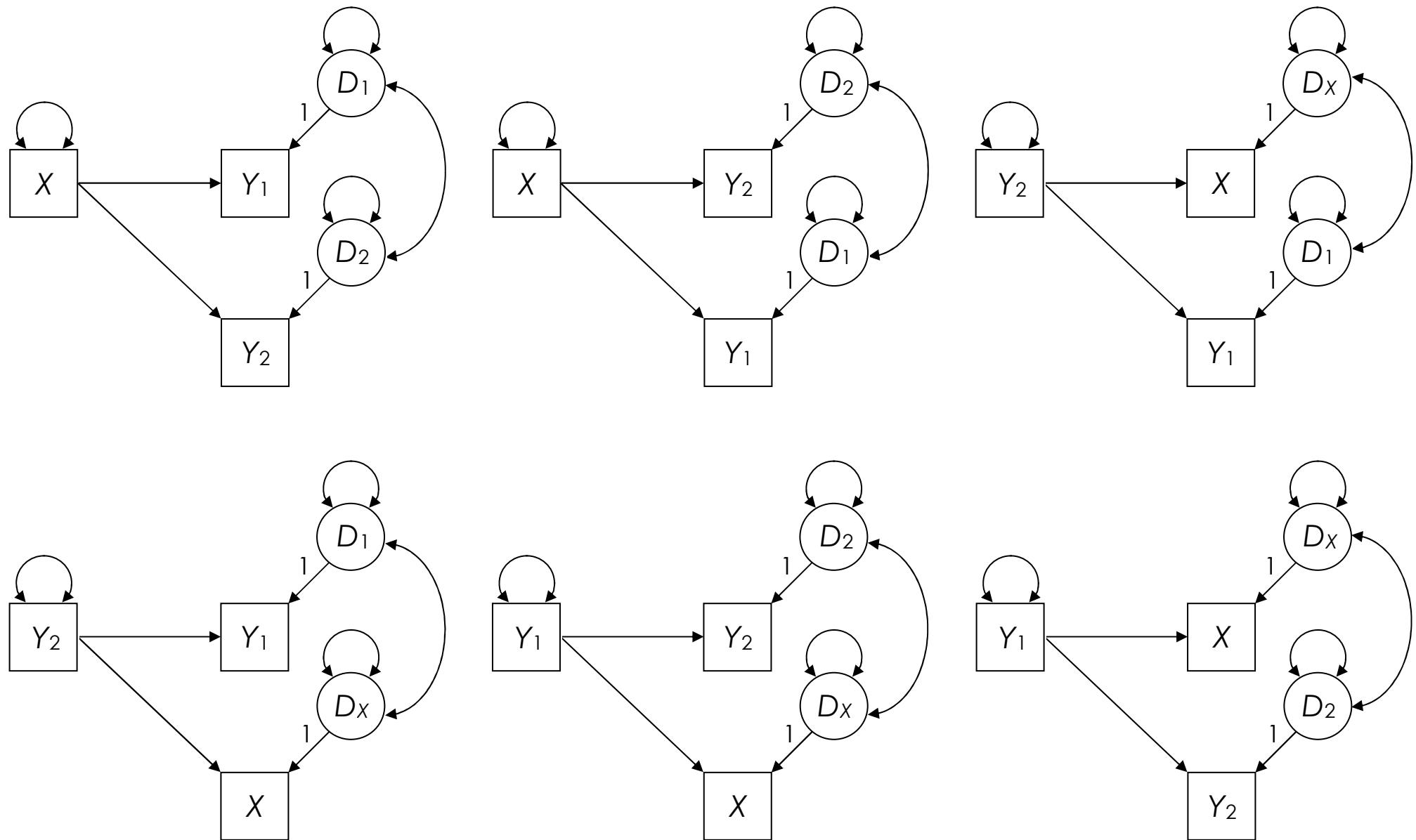
- Concurrent design
- Uncertain directionality
- Equivalent models

Concurrent measurement (1)



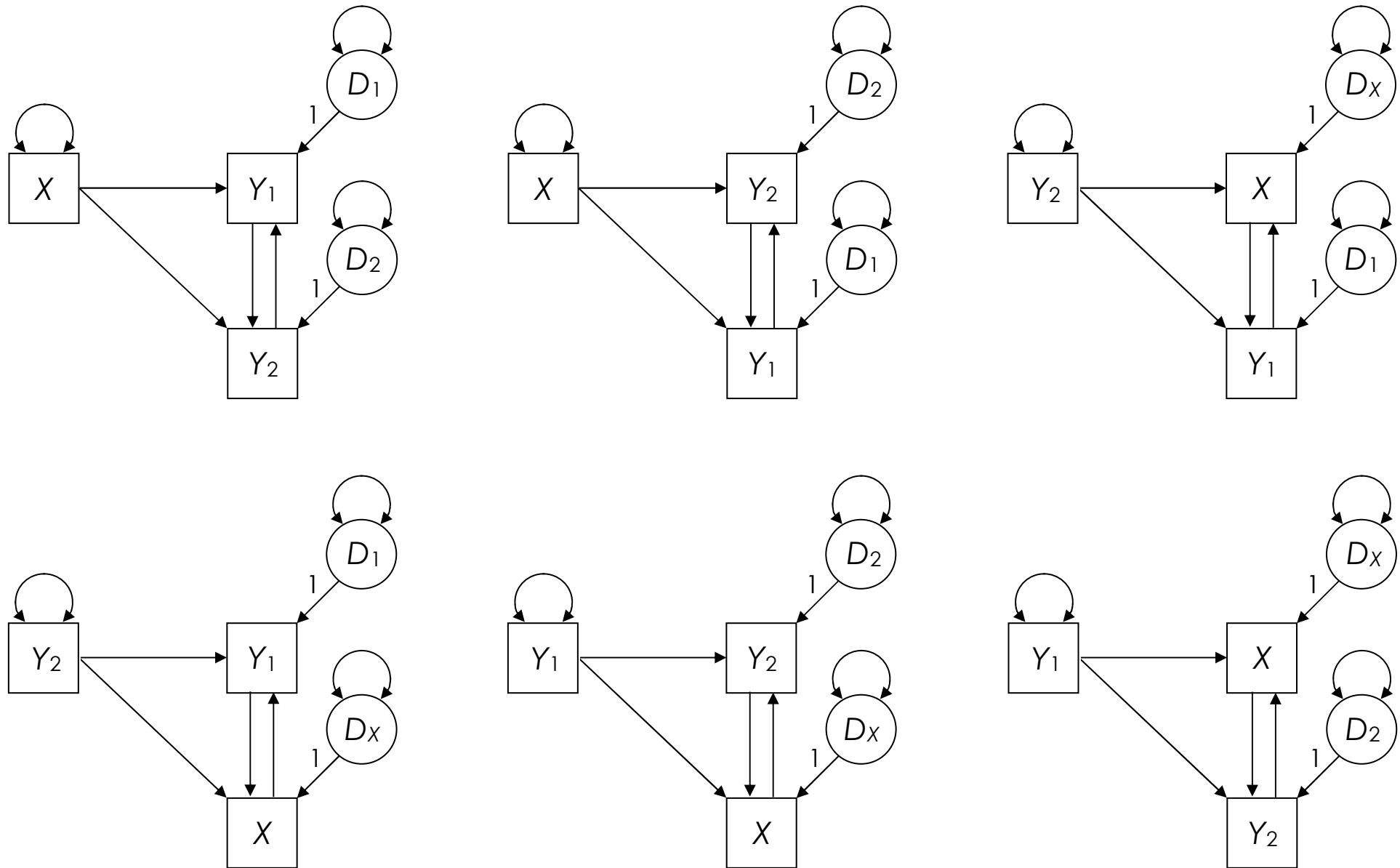
A12

Concurrent measurement (2)



A13

Concurrent measurement* (3)



*Equality-constrained reciprocal effects

A14

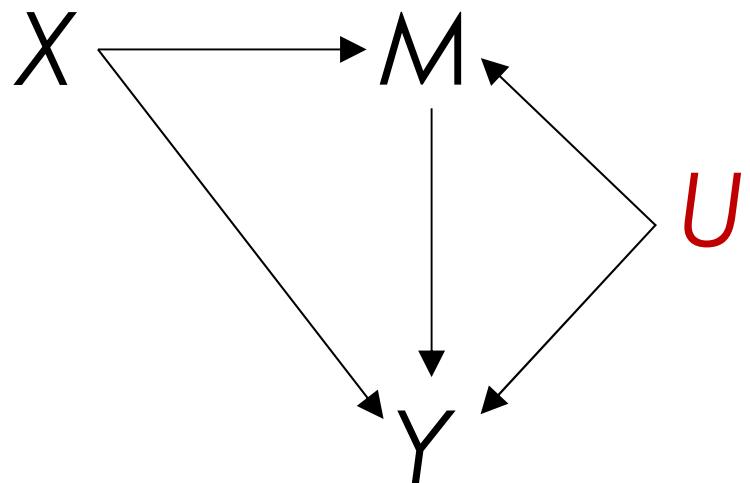
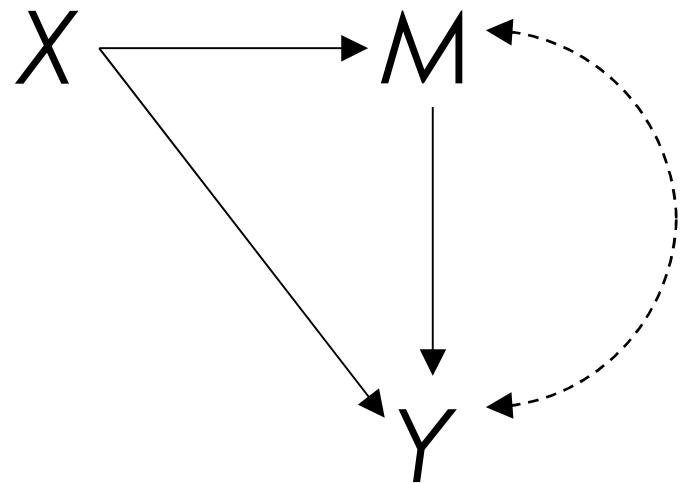
Design

- Minimal:

Time precedence

X is manipulated

M, Y are nonexperimental



$$M \longrightarrow Y$$

$$M \longleftarrow X \longrightarrow Y$$

$$M \longleftarrow U \longrightarrow Y$$

A16

Design

- Bullock, J. G., Green, D. P., & Ha, S. E. (2010). Yes, but what's the mechanism? (Don't expect an easy answer). *Journal of Personality and Social Psychology*, 98, 550–558.

Moderation

- X, W, Z are continuous
- Moderated MR
- $\hat{Y} = B_x X + B_w W + B_{xw} XW + A$

Moderation

- Edwards, J. R. (2009). Seven deadly myths of testing moderation in organizational research. In C. E. Lance & R. J. Vandenberg (Eds), *Statistical and methodological myths and urban legends: Doctrine, verity and fable in the organizational and social sciences* (pp. 143–164). New York: Taylor & Francis.

Myths

You **must** center, to reduce
extreme collinearity



Truths

Centering changes nothing

Optional, if 0 is not on scale

Center some, others not



Myth

You **must** use hierarchical entry



Truths

Not required

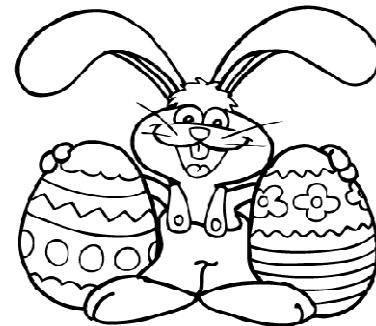
Possibly misleading



A23

Myth

You can ignore score
reliability



Truth

Score reliability is critical

$$r_{xx} > .90$$



Myth

$$\hat{Y} = B_x X + B_w W + B_{xw} XW + A$$

X, W are “main effects”



A26

Truth

X, W are linear only



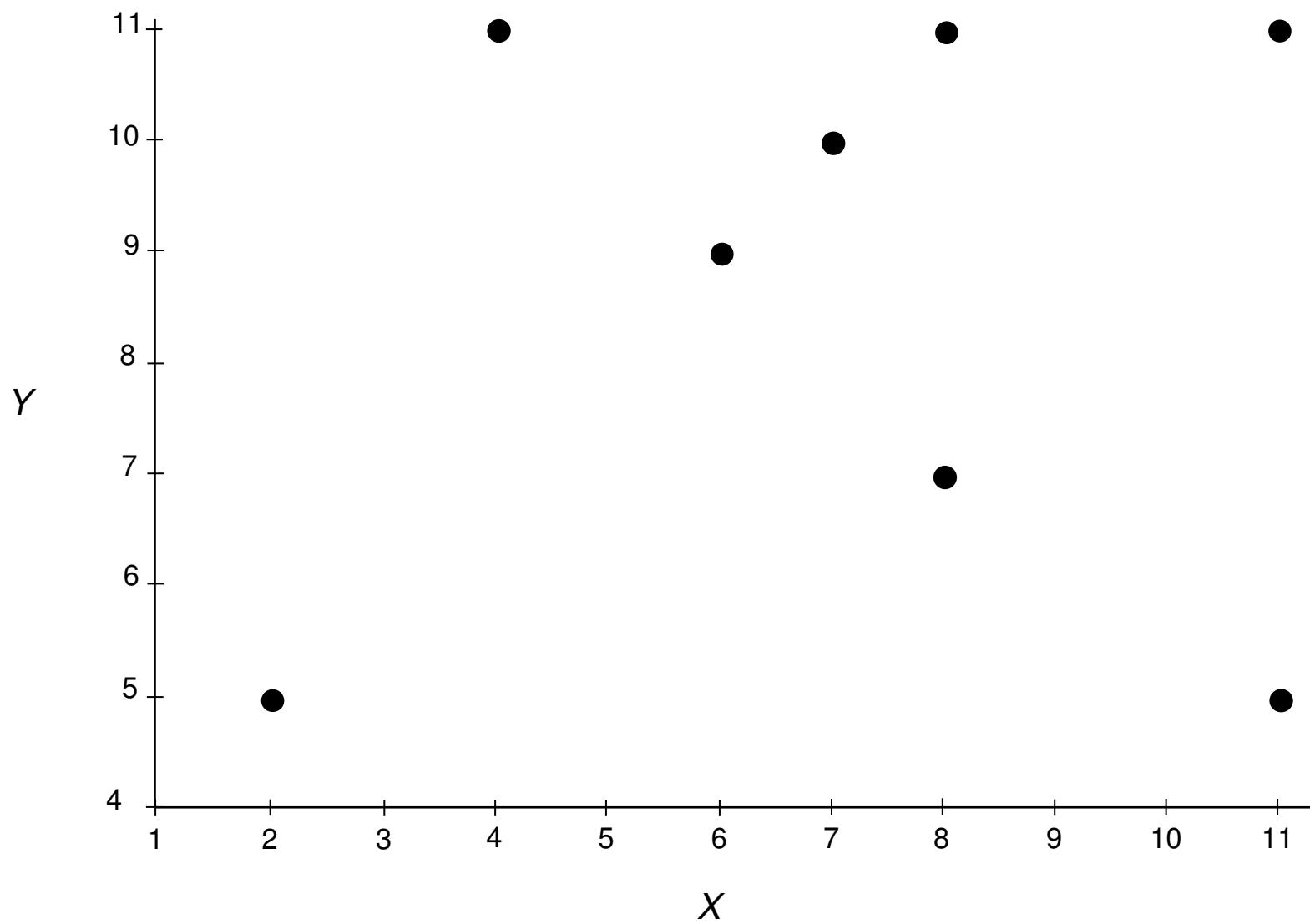
A27

X	W	Y
2	10	5
6	12	9
8	13	11
11	10	11
4	24	11
7	19	10
8	18	7
11	25	5
M	7.125	16.375

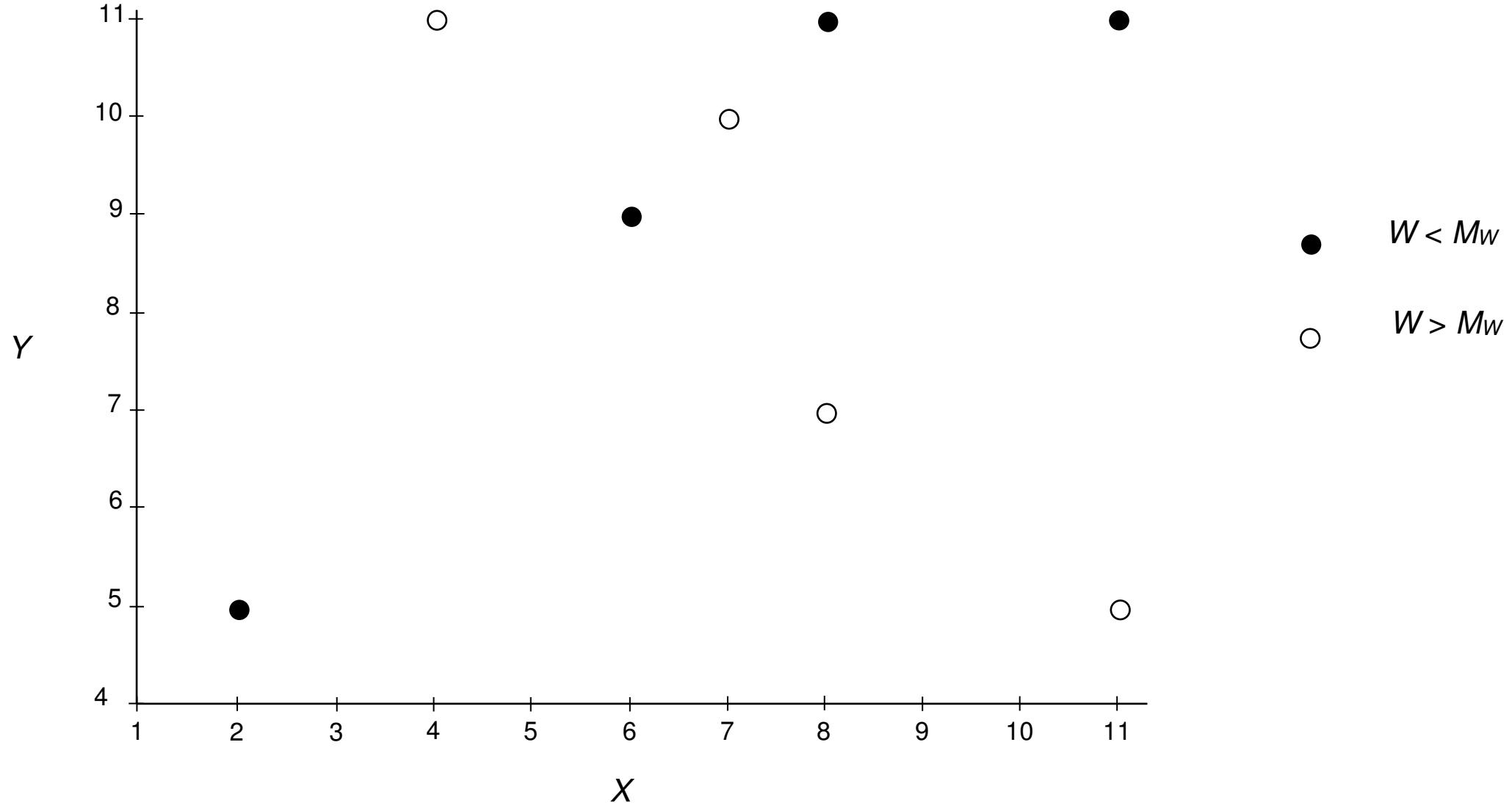
A28

$$\hat{Y} = .112X - .064W + 8.873$$

$$R^2 = .033$$



A30



A31

$$\hat{Y} = .112X - .064W + 8.873$$

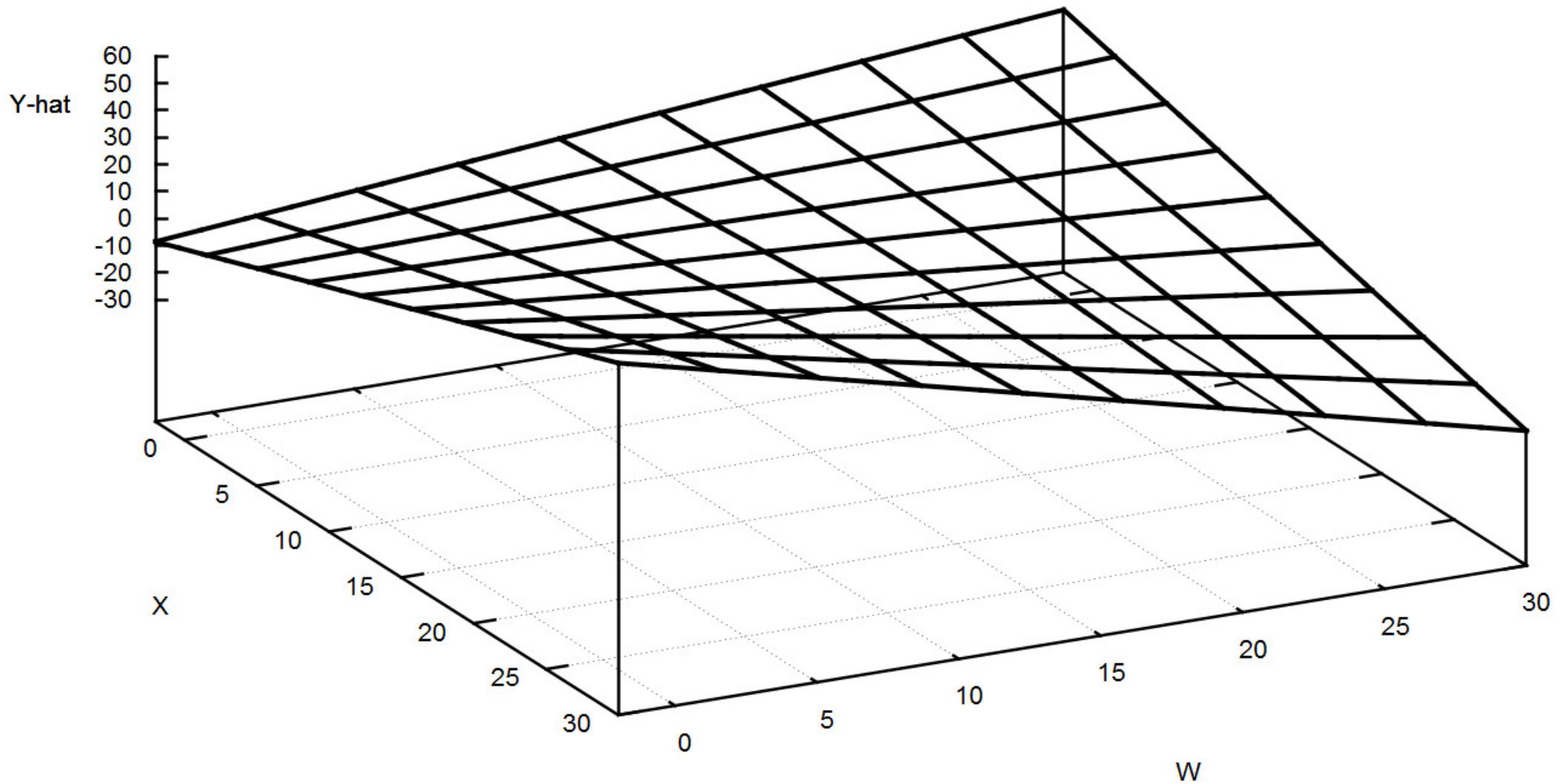
$$R^2 = .033$$

$$\hat{Y} = 1.768X + .734W - .108XW - 3.118$$

$$R^2 = .829$$

A32

$$\hat{Y} = 1.768X + .734W - .108XW - 3.118$$



Centering

- XW with X, W
- $x = X - M_x, w = w - M_w$
- xw with x, w

$$\hat{Y} = 1.768X + .734W - .108XW - 3.118$$

$$\hat{Y} = .000x - .035w - .108xw + 8.903$$

$$R^2 = .829$$

A35

Y on X as a function of W

$$\hat{Y} = 1.768X + .734W - .108XW - 3.118$$

$$\hat{Y} = 1.768X - .108XW + .734W - 3.118$$

$$\hat{Y} = (1.768 - .108W)X + (.734W - 3.118)$$

A36

$$\hat{Y} = (1.768 - .108W)X + (.734W - 3.118)$$

$$M_w = 16.38$$

4.34 10.36 16.38 22.40 28.42

$$\hat{Y} = (1.768 - .108W)X + (.734W - 3.118)$$

4.34, 10.36, 16.38, 22.40, and 28.42

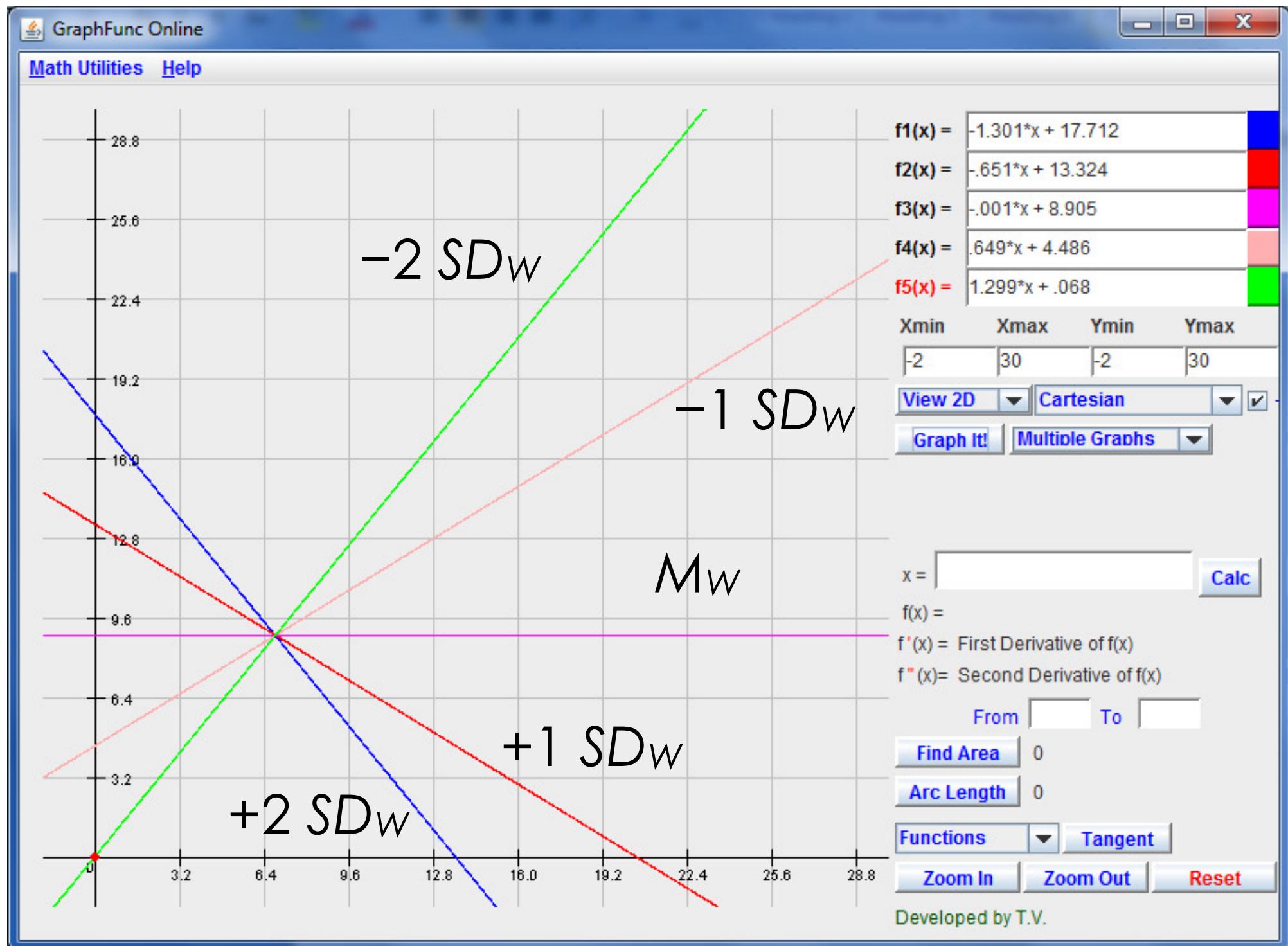


$$\hat{Y}_{W=22.40} = (1.768 - .108 * 22.40)X + (.734 * 22.40 - 3.118)$$

$$\hat{Y}_{W=22.40} = -.651X + 13.324$$

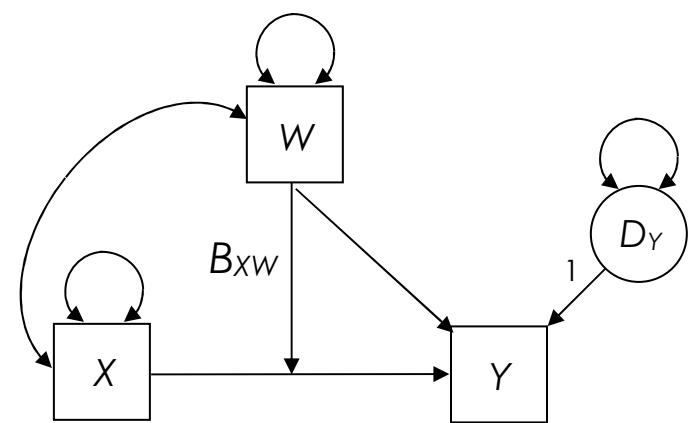
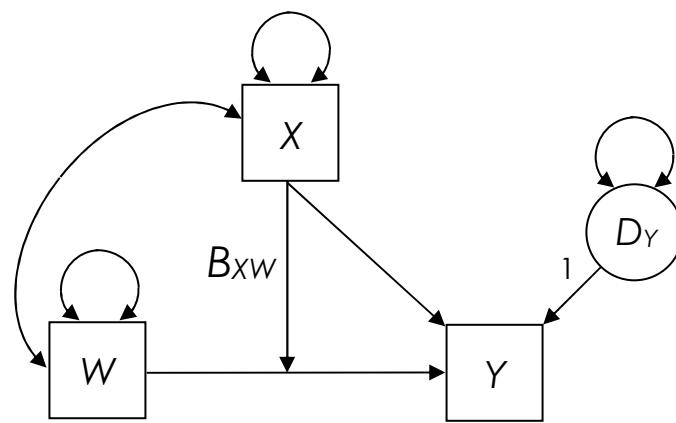
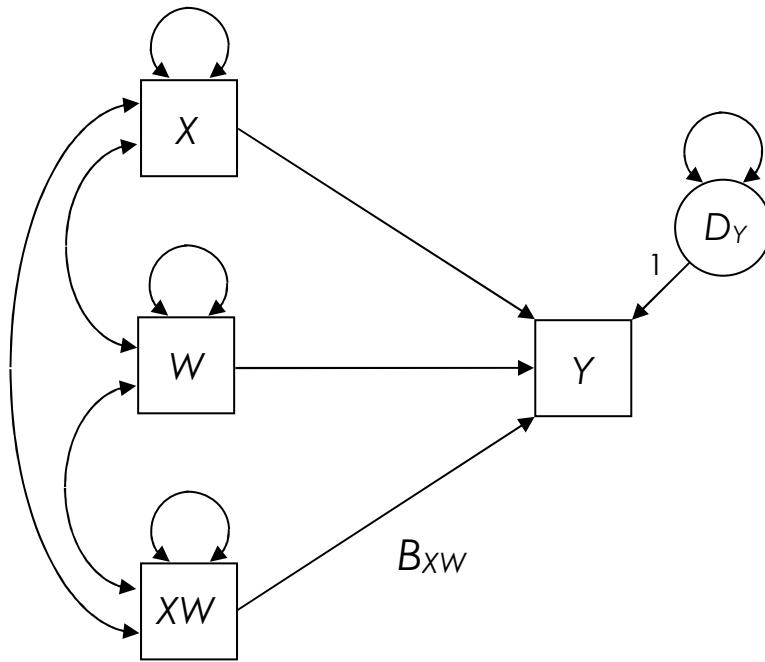
W

Level	Score	Regression equation
+2 SD	28.42	$\hat{Y} = -1.301X + 17.712$
+1 SD	22.40	$\hat{Y} = -.651X + 13.324$
Mean	16.38	$\hat{Y} = -.001X + 8.905$
-1 SD	10.36	$\hat{Y} = .649X + 4.486$
-2 SD	4.34	$\hat{Y} = 1.299X + .068$



Path models

- Diagram options:
 1. Include XW
 2. Moderated path

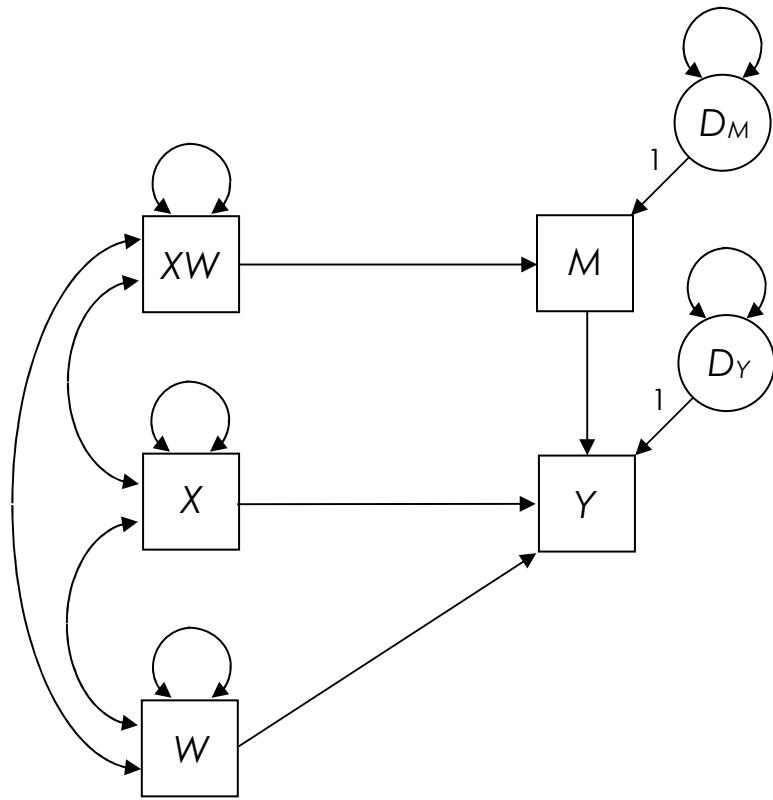


A42

Process modeling

- Both:
 1. Mediated moderation
 2. Moderated mediation
(conditional indirect)

Mediated moderation

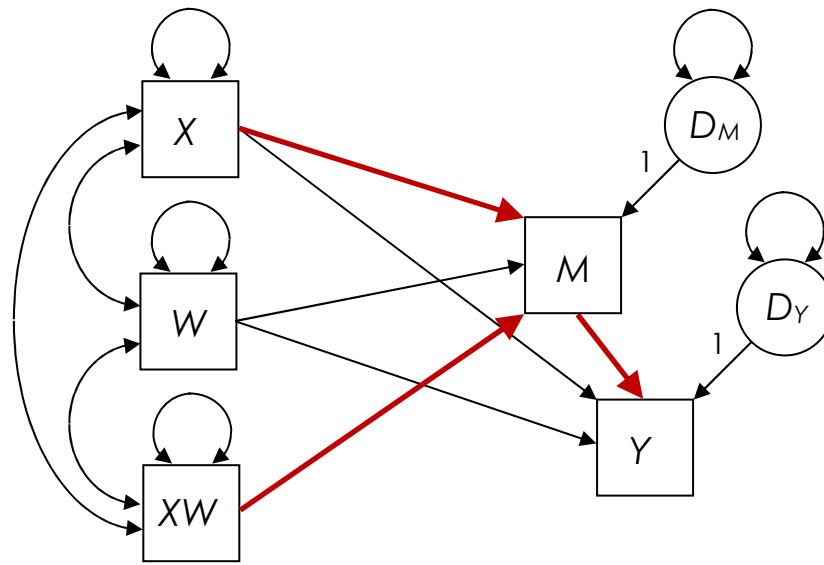


Process modeling

- Moderated mediation:
 1. 1st stage
 2. 2nd stage
 3. 1st and 2nd stage

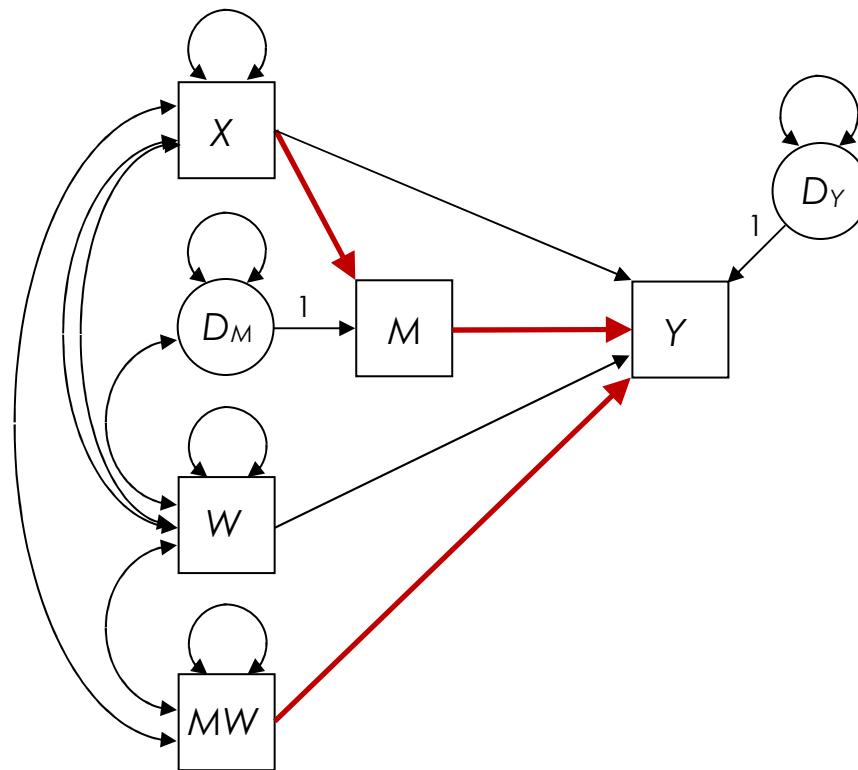
1st stage

$X \rightarrow M$ depends on W



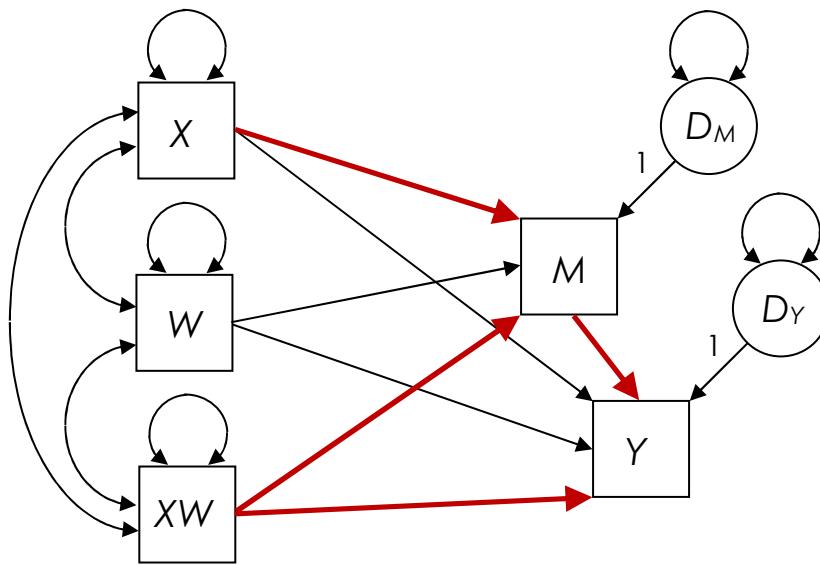
2nd stage

$M \rightarrow Y$ depends on W



1st and 2nd stage

$X \rightarrow M, M \rightarrow Y$ depend on W



Mediation in SCM

- Nonparametric model
- Consistent definition
- Linear or nonlinear models

Mediation in SCM

- Assumes $X \times M$:
 - Conditional direct, indirect
 - Equations, not diagrams

Mediation in SCM

- Assumes $X \times M$:
 - > 1 direct, indirect
- Counterfactuals (PO)

Mediation in SCM

- Direct effect:

- Controlled (CDE)

- Natural (NDE)

Mediation in SCM

- Total effect:

$$TE = NDE + NIE$$

Mediation in SCM

- CDE:

How much Y changes

Given $X = 0$ to $X = 1$

If $M = m$ for all cases

Mediation in SCM

- CDE:

Different value of DE

For every value of M

Mean Y change over M

Mediation in SCM

- NDE:

Allows for variation in M

But at values that would
be observed in control

Mediation in SCM

- NDE:

How much Y changes

Given $X = 0$ to $X = 1$

If M varies as under $X = 0$

Mediation in SCM

- NIE:

How much Y changes in $X = 1$

If M changes from values observed in $X = 0$ to values it would be obtained in $X = 1$

Counterfactuals

$$CDE = E[Y(X=1, M=m)] - E[Y(X=0, M=m)]$$

$$NDE = E[Y(X=1, M=m_0)] - E[Y(X=0, M=m_0)]$$

$$NIE = E[Y(X=1, M=m_1)] - E[Y(X=1, M=m_0)]$$

$$TE = E[Y(X=1)] - E[Y(X=0)]$$

Petersen, et al. (2006)

$X = 0$, control; $X = 1$, anti-viral therapy

M = viral load

Y = CD4 T-cells

A60

CDE

Mean Δ T-cells if viral load is same
for all cases

NDE

Mean Δ T-cells if viral load were
among untreated cases

A61

NIE

Mean Δ T-cells among treated if
viral load changed from untreated
to treated levels

A62

Mediation in SCM

- Valeri & VanderWeele (2013)
SAS/STAT, SPSS
- Muthén (2011)
Mplus

Mediation in SCM

- Hicks & Tingley (2012)
Stata
- Tingley et al. (2014)
R

thanks again