

## Mediation and Moderation Effects

### Moderation

Moderators and Mediators Together  
When Mediation Differs by Group  
Baseline by Treatment Interactions

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## Moderation Statements

- Treatment effects *differ for* males and females.
- Program effects on tobacco use *are greater for* people who are more likely to believe positive consequences of tobacco use at baseline.
- A program *works for* middle school students *but does not work for* high school students.
- Program effects *differ as a function of* baseline measures of the outcome variable.
- Success of nicotine patch treatment *differs depending on* whether person has a certain genetic disposition.

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## Definition of a Moderator (1)

- A moderator is a variable that affects the strength and/or form of the relation between X and Y
- Moderator variables determine for whom a treatment is effective when X represents assignment to a treatment group
- Moderator variables are often represented by the letter **Z**

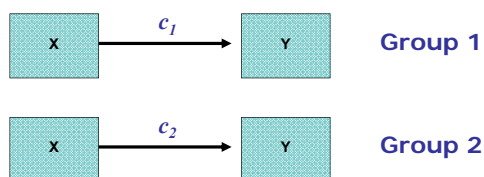
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## Definition of a Moderator (2)

- A moderator variable (Z) is not intermediate in the causal sequence between X and Y, so it is not a mediator variable (M).
- Moderator effects are also called interaction effects, such that the relation between X and Y depends on a third variable, the moderator (Z).

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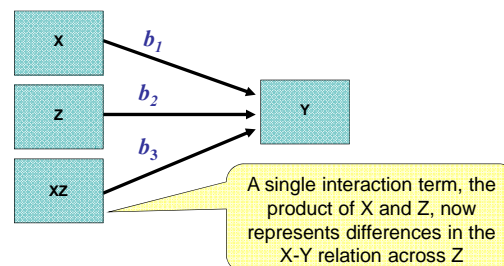
## Path Diagram of the Moderation Model for Individual Groups



Different regression coefficients predict Y from X in each group, indicating that the X-Y relation differs across the moderator

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## Path Diagram of the Moderation Model for Combined Groups



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## Testing Moderator Effects for Combined Groups

- Moderator effects are tested by including an interaction term to an equation that predicts Y from X, in addition to a main effect of Z

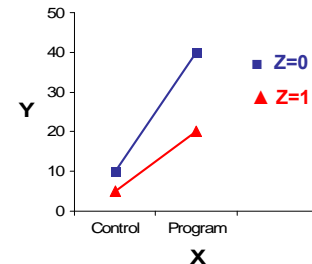
$$\hat{Y} = i_1 + c_1X + c_2Z + c_3XZ$$

- Lower order terms must be included in the equation for unbiased estimation of  $c_3$

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## Simple Slopes

- Graphing simple slopes shows how the effect of X on Y differs for Z = 0 and Z = 1



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## Centering Terms

- Centering terms (subtracting mean scores on a variable from each observed score) is important in moderation analysis to reduce multicollinearity and to adequately interpret regression coefficients
- The interaction term in the general moderation model is the product of the centered X and centered Z variables  
(see Aiken & West, 1991)

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## Why study both mediation and moderation effects?

- Both effects are important
  - Understand how manipulations achieve effects **and** identify characteristics of participants and/or environment that moderate effectiveness of a manipulation.
- Streamline/improve manipulations by understanding for whom and/or under what conditions they operate.
- Can test hypotheses regarding the consistency and specificity of results across groups.
- Better target subgroups by understanding how they differentially respond to manipulations
  - Does a program differentially affect participants based on level of risk?

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## Questions you can ask by Combining Mediation and Moderation Models (1)

- "Is the mechanism by which a manipulation achieves its effects the same across groups?"
  - Asks if the mediated effect differs across levels of a moderator variable  
(MODERATION OF THE MEDIATED EFFECT)
- "Is the reason an overall manipulation effect is moderated explained by a mediation process?"
  - Asks if an interaction effect can be explained by a mediating mechanism  
(MEDIATION OF A MODERATOR EFFECT)

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## Questions you can ask by Combining Mediation and Moderation Models (2)

- "Does the manipulation change the mediator in the same way across groups?"
  - Asks if the action theory of the manipulation is the same across levels of a moderator variable (TEST OF HOMOGENEITY IN THE *a* PATH)
- "Is the mediating variable related in the same way to the outcome across groups?"
  - Asks if the conceptual theory of the manipulation is the same across levels of a moderator variable (TEST OF HOMOGENEITY IN THE *b* PATH)

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### Moderators in and out of the Mediation Process

- **Moderator in the mediation process**, i.e., the mediating variable M or dependent variable Y, e.g., also larger effects for persons lower on the mediator or outcome.
- **Moderator out of the mediation process**, i.e., not X, M, or Y. There are different mediation relations at different values of the moderator, e.g., different effects for males and females.

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### The Interaction of X and M in the Single Mediator Model (Chapter 10)

- XM interaction test of whether the relation between M and Y differs across levels of X.
- Simple slopes and Simple Mediation Effects.
- A Fourth-variable Effect where XM is the fourth variable.

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### Three Mediation Equations

$$Y = i_1 + cX + e_1$$

$$Y = i_2 + c'X + bM + hXM + e_2$$

$$M = i_3 + aX + e_3$$

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### Assumption of no XM Interaction

- An assumption of the single mediator model without the XM interaction is that the M to Y relation was the same across levels of X, i.e., the  $b$  path was equal across levels of X.
  - If the  $b$  path differs it means that the conceptual theory relation differs at different values of X.
  - The assumption can be tested by including the XM interaction in the model where both X and M predict Y. If it is nonsignificant, the evidence is that the  $b$  paths do not differ.
- There are cases where  $b$  is expected to differ
  - Example: a drug prevention program targets skills to deal with drug offers so that the relation between offers and drug use is much less for participants receiving the program

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### The XM Interaction (Equation 10.3)

$$Y = i_2 + c'X + bM + hXM + e_2$$

- The  $h$  coefficient represents whether the  $b$  path differs across levels of X (Judd & Kenny, 1981).
- If  $h$  is statistically significant it means there may be a more complicated form of mediation where the  $b$  path differs across groups.

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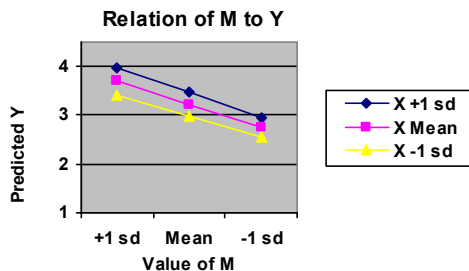
### Test of the XM Interaction for the Water Consumption Example

$$Y = i_2 + c'X + bM + hXM + e_2$$

- $h = .0299$ ,  $s_h = .1198$ ,  $t_h = 0.25$  so there is not evidence that the relation of M to Y differs across the two groups for the example described earlier.

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## Plot of the XM interaction for the Water Consumption Example



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## Simple Mediation Effects #1

- A significant XM interaction means that the  $b$  path differs across levels of  $X$ .
- The water consumption plot in the last slide showed the different  $b$  value for +1SD, mean and -1SD values of  $X$ . These are simple slopes. Remember  $X$  was continuous.
- A simple mediation effect would be the value of  $ab$  at different values of  $X$ , e.g., the simple mediation effect at the mean.
- The standard error of the simple mediation effect uses the  $a$  coefficient and standard error and  $b$  coefficient and standard error--at a certain value of  $X$ .

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## Simple Mediation Effects #2

- For a binary  $X$ , there are two simple slopes, e.g., treatment and control. For a continuous  $X$ , there are many simple slopes and simple mediated effects and a different mediated effect at the different values of  $X$ .
- The significance of the  $b$  path is obtained by centering the  $X$  variable at different values and the significance of  $b$  is obtained from the corresponding statistical analysis.
- If  $X$  is centered at zero, then the  $b$  path significance test is at an  $X$  value of 0. If  $X$  is centered so its average is 1SD above the mean then the significance of the  $b$  path is the value in the output. This can be done for any value of  $X$  to test simple slopes and simple mediation effects.

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## When XM interactions occur?

1. Measurement. The mediating variable means something different in the two groups.
2. Non-linear relation between  $M$  and  $Y$ . The  $X$  intervention changes the level of  $M$  so that the relation between  $M$  and  $Y$  in the program group differs.
3. Restriction of range.  $X$  changes  $M$  to a level where there is a ceiling or floor effect so the relation is not as large.
4. Longitudinal. There is change in  $M$  in the experimental group but no change in the control group.
5. Omitted Variable. There is an omitted variable that comes into play at different values of  $M$ .

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## Example XM interactions 1.

- 1. Dietary intervention ( $X$ ) teaches knowledge of healthy diet ( $M$ ) which is expected to improve diet. Without intervention, dietary behavior results from habit, not knowledge. Control group has a low relation between knowledge and diet behavior. In intervention group, the relation between knowledge and diet is stronger because participants learn about diet (Judd & Kenny, 1981).
- 2. Mindfulness intervention increases attention to pain. In the control group attention to pain increases experience of pain. In the intervention group, attention to pain reduces pain because of the mindfulness intervention.

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## Example XM interactions 2.

- 3. Intervention ( $X$ ) teaches social competence ( $M$ ) to reduce aggressiveness. For persons low on social competence, the program effect is much larger than for persons already high on social competence.
- 4. Intervention ( $X$ ) increases self-efficacy to eat properly which improves diet ( $Y$ ). For persons whose diet is already appropriate, the program does not have much of an effect.

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## Moderators of Mediation Relations as a Fourth variable, Z

- X, M, and Y are measured and a fourth variable Z, the moderator, is now included in the model.
  - There are many different types of relations in a model that contains X, M, Y, and Z.
- The moderator is usually a variable across which mediation relations differ, not a variable that causes X, M, or Y, but it could also be a cause of these variables.
- Examples of moderators: (1) Stable: gender, age, race, (2) Individual Differences: SES, risk taking, impulsivity

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## Examining Mediation and Moderation for Individual Groups

- By testing the mediation model for different groups we can examine several possibilities:
  - Homogeneity of the Mediated Effect (Question 1)
  - Homogeneity of Action Theory (Question 3)
  - Homogeneity of Conceptual Theory (Question 4)

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## Homogeneity of Action Theory for Individual Groups

$$H_0: a_{group1} - a_{group2} = 0$$

$$H_1: a_{group1} - a_{group2} \neq 0$$

- Heterogeneous action theory corresponds to different **a** paths (i.e., the first link in the mediation model) across moderator-based groups

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## Testing Homogeneous Action Theory for Individual Groups

- A significance test for the effect is computed by taking the difference of the **a** paths across groups and dividing the estimate by a standard error of the difference:

$$\frac{\hat{a}_1 - \hat{a}_2}{\sqrt{s_{\hat{a}_1}^2 + s_{\hat{a}_2}^2}}$$

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## Homogeneity of Conceptual Theory for Individual Groups

$$H_0: b_{group1} - b_{group2} = 0$$

$$H_1: b_{group1} - b_{group2} \neq 0$$

- Heterogeneous conceptual theory corresponds to different **b** paths (i.e., the second link in the mediation model) across moderator-based groups

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## Testing Homogeneous Conceptual Theory for Individual Groups

- A significance test for the effect is computed by taking the difference of the **b** paths across groups and dividing the estimate by a standard error of the difference:

$$\frac{\hat{b}_1 - \hat{b}_2}{\sqrt{s_{\hat{b}_1}^2 + s_{\hat{b}_2}^2}}$$

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## Homogeneity of the Mediated Effect for Individual Groups

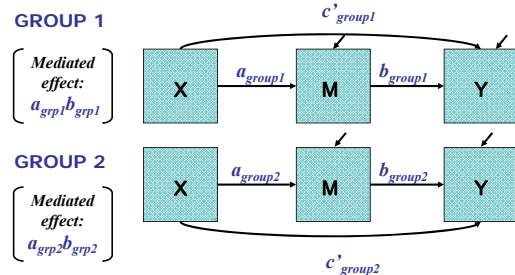
$$H_0: a_{\text{group1}}b_{\text{group1}} - a_{\text{group2}}b_{\text{group2}} = 0$$

$$H_1: a_{\text{group1}}b_{\text{group1}} - a_{\text{group2}}b_{\text{group2}} \neq 0$$

- A heterogeneous mediated effect corresponds to moderation of the mediated effect (has been called moderated mediation)

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## Path Model for Testing Homogeneity of Mediated Effect in Individual Groups



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## Book Example for Mediation and Moderation for Individual Groups

- Chapter 3 Water Consumption example:
  - A variable Z was introduced into study, creating two groups:
    - Group Z = 0: Normal Participants (Chapter 3)
    - Group Z = 1 Fit Participants (Chapter 10)
- Recall X = temperature, M = self-reported thirst, Y = water consumed.
- Do the two groups differ in how self-reported thirst mediates the relation of temperature on water consumption?

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## Book Example for Mediation and Moderation for Individual Groups

$$\frac{\hat{a}\hat{b}_{\text{group1}} - \hat{a}\hat{b}_{\text{group2}}}{\sqrt{s^2_{a1b1} + s^2_{a2b2}}} = \frac{.15272 - .26448}{\sqrt{.005489 + .016096}} = -.76069$$

- There is not significant moderation of the mediated effect. That is, the mechanism by which temperature affects water consumption is the same across normal and fit participants.
- Note that assuming the two groups are independent, the standard error of this test is the pooled standard error of the mediated effect from each group.

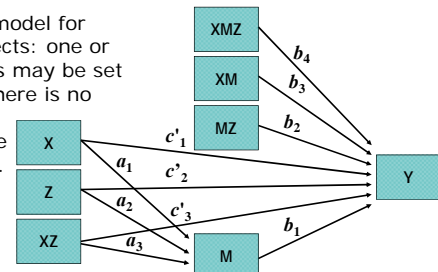
## Examining Mediation and Moderation for Combined Groups

- As in the basic moderation model, moderator effects in the mediation model may be represented for combined groups
- There will be two equations for the combined group notation because there are two equations in the basic mediation model
- Interaction terms in the equations will now represent the group differences as with the basic moderation model

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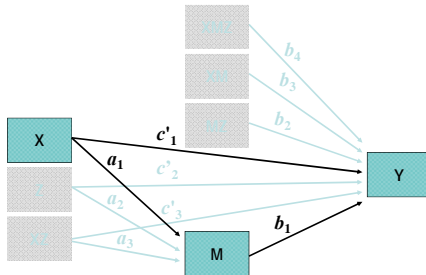
## Mediation and Moderation for Combined Groups

A general model for testing effects: one or more terms may be set to zero if there is no reason to hypothesize their effect.



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## Mediation in the General Model for Testing Mediation & Moderation



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## Mediation and Moderation for Combined Groups Hypotheses (1)

- Test of homogenous action theory is now:

$$H_0: a_3 = 0$$

$$H_1: a_3 \neq 0$$

- $(a_3 = 0)$  is equivalent to  $(a_{group1} - a_{group2} = 0)$  when the moderator is dichotomous

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## Mediation and Moderation for Combined Groups Hypotheses (2)

- Test of homogenous conceptual theory is now:

$$H_0: b_2 = 0$$

$$H_1: b_2 \neq 0$$

- $(b_2 = 0)$  is equivalent to  $(b_{group1} - b_{group2} = 0)$  when the moderator is dichotomous

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## Mediation and Moderation for Combined Groups Hypotheses (3)

- Test of a homogenous mediated effect is more complicated to test
- Some argue that a joint significance test of  $a_3$  and  $b_2$  can provide evidence for moderation of the mediated effect

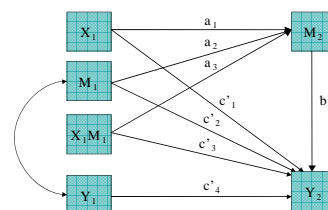
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## An Example XM interaction: Baseline by Treatment Interaction

- Mediation depends on the baseline measure of the mediating variable
- Program effects are often largest for persons with the lowest scores on the mediator at baseline
- Baseline levels of the mediator ( $M_1$ ) act as a moderator variable
- Two waves of data are needed for this design, such that X predicts  $M_2$  which predicts  $Y_2$ , with  $M_1$  acting as a moderator of the relation

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## Baseline by Treatment Interaction Path Diagram (Morgan-Lopez, 2003)



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## Summary of Analyzing Mediation and Moderation Together

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- Mediation and moderator effects can be analyzed simultaneously in the same model.
- Both mediation and moderation are important for investigating how programs work. Can test homogeneous action and conceptual theory across subgroups.
- Moderators can be inside or outside the mediation process.
- Models are available to test different effects of interest when jointly analyzing mediation and moderation.

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