Multiple Mediator Model Example - - SAS, MPlus, and SPSS Handouts

This example is based on Harris and Rosenthal's (1985) four-mediator model for how teacher expectancies affect student achievement. Two mediators were examined in this study, social climate, a measure of verbal and nonverbal warmth, and input, the tendency to teach more material and more difficult material to high-expectancy students.

The data for the 40 subjects in this hypothetical study of teacher expectancies and student achievement are shown below where X is the teacher expectancy based on an intelligence test given to the student the previous year, M1 is the average observer rating of social warmth, M2 is the average observer rating of input to the student, and Y is the score on the test at the end of the semester.

Data for hypothetical study of teacher expectations and student achievement (Mackinnon, 2008, pp.113):

S#	X	M1	M2	Y	S#	X	M1	M2	Y
1	51	41	54	59	21	53	69	44	84
2	40	34	51	60	22	53	67	40	82
3	55	42	53	60	23	40	49	45	74
4	35	22	56	61	24	34	40	37	62
5	47	34	45	47	25	32	40	49	54
6	58	52	79	84	26	56	60	51	81
7	56	57	55	69	27	55	46	65	89
8	53	49	55	85	28	51	58	54	83
9	38	42	46	75	29	50	53	56	75
10	73	80	48	87	30	45	61	52	72
11	57	42	65	85	31	63	42	40	63
12	54	62	55	73	32	46	39	51	69
13	68	54	55	77	33	60	62	53	66
14	46	41	62	50	34	48	41	56	72
15	48	44	43	58	35	46	40	46	68
16	56	54	54	69	36	50	51	52	73
17	67	73	61	99	37	49	51	55	69
18	47	61	38	64	38	35	39	46	46
19	60	59	42	65	39	50	44	46	70
20	54	51	55	68	40	47	40	68	76

SAS

Syntax

```
* The following program code reads in the dataset for the two mediator model
example in MacKinnon 2008, Chapter 5. Five variables are defined: (a) subject
ID#, (b) teacher expectancy, i.e., X, (c) social climate, i.e., M1, (d)
input, i.e., M2, and (e) test score at the end of the semester, i.e., Y;

data a;
input s x m1 m2 y;
datalines;
    1    51      41       54      59
    2    40      34       51      60
.
.
.  ;

* The following commands run the four mediation regression equations outlined
  in Chapter 5 for two-mediator model example;

/* Equation 5.1 */

proc reg;
model y=x;

/* Equation 5.2. Note "covb" gives the covariance of the regression
coefficients. This value is needed for computing standard errors*/

model y=x m1 m2/covb;

/* Equation 5.3 */

model m1=x;

/* Equation 5.4 */

model m2=x;
run;
```

NOTE:

There is one total effect and one direct effect, but two mediated effects in the model. This is why there are two *b* paths and two *a* paths, but only one *c* & one *c'*.

SAS Output:

```
The REG Procedure
Model: MODEL1
Dependent Variable: y
```

Parameter Estimates

	Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
	Intercept	1	34.72693	8.92472	3.89	0.0004
$\hat{c}$ ⇨	x	1	0.70776	0.17343	4.08	0.0002

```
The REG Procedure
Model: MODEL2
Dependent Variable: y
```

Parameter Estimates

	Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
	Intercept	1	9.12333	10.47839	0.87	0.3897
$\hat{c}'$ ⇨	x	1	0.11215	0.20731	0.54	0.5919
$\hat{b}_1$ ⇨	m1	1	0.56903	0.15681	3.63	0.0009
$\hat{b}_2$ ⇨	m2	1	0.52972	0.16964	3.12	0.0035

Covariance of Estimates

Variable	Intercept	x	m1	m2
Intercept	109.79673207	-0.387895193	-0.495870555	-1.227545009
x	-0.387895193	0.0429780463	-0.022410629	-0.013017423
m1	-0.495870555	-0.022410629	0.02459002	0.0078936364
m2	-1.227545009	-0.013017423	$\boxed{0.0078936364}$	0.0287768709

$S_{\hat{b}_1\hat{b}_2}$

NOTE:

Although a single equation estimates both *b* paths (MODEL2), two models are needed to estimate the *a* paths (MODELS 3 and 4).

```
The REG Procedure
Model: MODEL3
Dependent Variable: m1
```

Parameter Estimates

	Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
	Intercept	1	7.09702	8.12853	0.87	0.3881
$\hat{a}_1 \Rightarrow$	x	1	0.84014	0.15796	5.32	<.0001

```
The REG Procedure
Model: MODEL4
Dependent Variable: m2
```

Parameter Estimates

	Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
	Intercept	1	40.71060	7.51397	5.42	<.0001
$\hat{a}_2 \Rightarrow$	x	1	0.22190	0.14602	1.52	0.1369

Computation of Standard Errors for Specific & Total Mediated Effects

$$s_{\hat{a}_1\hat{b}_1} = \sqrt{\hat{a}_1^2 s_{\hat{b}_1}^2 + \hat{b}_1^2 s_{\hat{a}_1}^2}$$
$$= \sqrt{(.84014^2)(.15681^2) + (.56903^2)(.15796^2)}$$
$$= .15948$$

$$s_{\hat{a}_2\hat{b}_2} = \sqrt{\hat{a}_2^2 s_{\hat{b}_2}^2 + \hat{b}_2^2 s_{\hat{a}_2}^2}$$
$$= \sqrt{(.22190^2)(.16964^2) + (.52972^2)(.14602^2)}$$
$$= .08602$$

Equation 5.5

Standard errors for the two, *specific*, mediated effects.

$$s_{\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2} = \sqrt{s_{\hat{a}_1}^2 \hat{b}_1^2 + s_{\hat{b}_1}^2 \hat{a}_1^2 + s_{\hat{a}_2}^2 \hat{b}_2^2 + s_{\hat{b}_2}^2 \hat{a}_2^2 + 2\hat{a}_1\hat{a}_2 s_{\hat{b}_1\hat{b}_2}}$$
$$= \sqrt{\begin{aligned}&(.15796^2)(.56903^2) + (.15681^2)(.84014^2) + (.14602^2)(.52972^2) + \\ &(.16964^2)(.22190^2) + 2(.84014)(.22190)(.00789)\end{aligned}}$$
$$= .18915$$

Equation 5.7

Standard error for the *total* mediated effect.

Testing Significance (i.e., computing *z*-scores) of Specific & Total Mediated Effects

$$\frac{\hat{a}_1\hat{b}_1}{s_{\hat{a}_1\hat{b}_1}} = \frac{.47806}{.15948} = 2.99765,\ p < .05 \qquad \frac{\hat{a}_2\hat{b}_2}{s_{\hat{a}_2\hat{b}_2}} = \frac{.11754}{.08602} = 1.33648, ns$$

$$\frac{\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2}{s_{\hat{a}_1\hat{b}_1+\hat{a}_2\hat{b}_2}} = \frac{.47806 + .11754}{.18915} = 3.14882,\ p < .05$$

Normal Theory Confidence Limits for Each Specific Mediated Effect

a_1b_1:

$$LCL_1 = \hat{a}_1\hat{b}_1 - 1.96(s_{\hat{a}_1\hat{b}_1}) = .47806 - (1.96 * .15948) = .16548$$

$$UCL_1 = \hat{a}_1\hat{b}_1 + 1.96(s_{\hat{a}_1\hat{b}_1}) = .47806 + (1.96 * .15948) = .79065$$

significant, because zero is not contained in the interval

a_2b_2:

$$LCL_2 = \hat{a}_2\hat{b}_2 - 1.96(s_{\hat{a}_{21}\hat{b}_2}) = .11754 - (1.96 * .08602) = -.05106$$

$$UCL_2 = \hat{a}_2\hat{b}_2 + 1.96(s_{\hat{a}_2\hat{b}_2}) = .11754 + (1.96 * .08602) = .28615$$

non-significant, because zero is contained in the interval

Asymmetric Confidence Limits for Each Specific Mediated Effect

a_1b_1: LCL_1 = .19889
UCL_1 = .82199

a_2b_2: LCL_2 = -.02648
UCL_2 = .31099

Are the two mediated effects significantly different from each other?

$$s_{\hat{a}_1\hat{b}_1-\hat{a}_2\hat{b}_2} = \sqrt{s^2_{\hat{a}_1}\hat{b}_1^2 + s^2_{\hat{b}_1}\hat{a}_1^2 + s^2_{\hat{a}_2}\hat{b}_2^2 + s^2_{\hat{b}_2}\hat{a}_2^2 - 2\hat{a}_1\hat{a}_2 s_{\hat{b}_1\hat{b}_2}}$$

SE of estimate:

$$= \sqrt{\begin{aligned}&(.15796^2)(.56903^2) + (.15681^2)(.84014^2) + (.14602^2)(.52972^2) + \\ &(.16964^2)(.22190^2) - 2(.84014)(.22190)(.00789)\end{aligned}}$$

$$= .1729$$

Standard error of the difference between the two mediated effects.

z-statistic:

$$\frac{estimate}{se_{estimate}} = \frac{\hat{a}_1\hat{b}_1 - \hat{a}_2\hat{b}_2}{s_{\hat{a}_1\hat{b}_1-\hat{a}_2\hat{b}_2}} = \frac{.47806 - .11754}{.1729} = 2.0852,\ p < .05$$

MPLUS

This output is annotated for your review. Note that although the TECH3 output option used in this example provides you with both covariance and correlation matrices of the parameter estimates, only the covariance matrix is provided for you here to conserve space.

Syntax:

```
Title: CHAPTER 5 Two Mediator Model Example;

Data:
Nobs=40;
File is E:/Chap5_twomed.txt;

Variable:
Names are ID X M1 M2 Y;
Usevariables are X M1 M2 Y;

Analysis:
Type is general;
Estimator is ML;

Model:
Y on M1 M2 X;
M1 on X;
M2 on X;
M1 with M2;
X;

Model Indirect:
Y ind X;

Output:
cinterval tech1 tech3 ;
```

"tech3" gives you a covariance matrix of the parameters. **"tech1"** shows you numbering of the parameters

MPlus Output:

```
MODEL RESULTS

                    Estimates     S.E.  Est./S.E.


 Y        ON

    M1                  0.569    0.149      3.825

    M2                  0.530    0.161      3.292

    X                   0.112    0.197      0.570

 M1       ON

    X                   0.840    0.154      5.457

 M2       ON

    X                   0.222    0.142      1.559
 M1       WITH
    M2                -21.515   11.958     -1.799

 Variances
    X                  82.728   18.498      4.472

 Residual Variances
    M1                 78.436   17.539      4.472
    M2                 67.024   14.987      4.472
    Y                  63.321   14.159      4.472
```

Annotations beside the rows above: Y ON M1 $(\hat{b}_1)$; Y ON M2 $(\hat{b}_2)$; Y ON X $(\hat{c}')$; M1 ON X $(\hat{a}_1)$; M2 ON X $(\hat{a}_2)$.

```
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                    Estimates     S.E.  Est./S.E.

Effects from X to Y

  Total                 0.708    0.169      4.187


  Total indirect        0.596    0.170      3.499

  Specific indirect

    Y
    M1
    X                   0.478    0.153      3.132

    Y
    M2
    X                   0.118    0.083      1.409
```

Annotations beside the rows above: Total $(\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2) + \hat{c}'$; Total indirect $(\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2)$; Specific indirect via M1 $(\hat{a}_1\hat{b}_1)$; via M2 $(\hat{a}_2\hat{b}_2)$.

```
  Direct
    Y
    X                   0.112    0.197      0.570
```
$(\hat{c}')$

```
CONFIDENCE INTERVALS OF MODEL RESULTS

                   Lower .5%  Lower 2.5%  Estimates  Upper 2.5%  Upper .5%

 Y        ON
    M1                 0.186       0.277      0.569       0.861      0.952
    M2                 0.115       0.214      0.530       0.845      0.944
    X                 -0.394      -0.273      0.112       0.498      0.619

 M1       ON
    X                  0.444       0.538      0.840       1.142      1.237

 M2       ON
    X                 -0.145      -0.057      0.222       0.501      0.588
 M1       WITH
    M2               -52.317     -44.953    -21.515       1.923      9.287

 Variances
    X                 35.079      46.471     82.728     118.984    130.376

 Residual Variances
    M1                33.259      44.060     78.436     112.812    123.612
    M2                28.420      37.649     67.024      96.398    105.627
    Y                 26.850      35.569     63.321      91.072     99.791

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                   Lower .5%  Lower 2.5%  Estimates  Upper 2.5%  Upper .5%


Effects from X to Y

  Total                0.272       0.376      0.708       1.039      1.143
  Total indirect       0.157       0.262      0.596       0.929      1.034
  Specific indirect

    Y
    M1
    X                  0.085       0.179      0.478       0.777      0.871

    Y
    M2
    X                 -0.097      -0.046      0.118       0.281      0.332
  Direct
    Y
    X                 -0.394      -0.273      0.112       0.498      0.619
```

TECHNICAL 1 OUTPUT

PARAMETER SPECIFICATION

LAMBDA

	M1	M2	Y	X
M1	0	0	0	0
M2	0	0	0	0
Y	0	0	0	0
X	0	0	0	0

THETA

	M1	M2	Y	X
M1	0			
M2	0	0		
Y	0	0	0	
X	0	0	0	0

BETA

	M1	M2	Y	X
M1	0	0	0	1
M2	0	0	0	2
Y	3	4	0	5
X	0	0	0	0

PSI

	M1	M2	Y	X
M1	6			
M2	7	8		
Y	0	0	9	
X	0	0	0	10

STARTING VALUES

LAMBDA

	M1	M2	Y	X
M1	1.000	0.000	0.000	0.000
M2	0.000	1.000	0.000	0.000
Y	0.000	0.000	1.000	0.000
X	0.000	0.000	0.000	1.000

THETA

	M1	M2	Y	X
M1	0.000			
M2	0.000	0.000		
Y	0.000	0.000	0.000	
X	0.000	0.000	0.000	0.000

BETA

	M1	M2	Y	X
M1	0.000	0.000	0.000	0.000
M2	0.000	0.000	0.000	0.000
Y	0.000	0.000	0.000	0.000
X	0.000	0.000	0.000	0.000

PSI

	M1	M2	Y	X
M1	70.168			
M2	0.000	36.460		
Y	0.000	0.000	69.741	
X	0.000	0.000	0.000	42.424

TECHNICAL 3 OUTPUT

ESTIMATED COVARIANCE MATRIX FOR PARAMETER ESTIMATES

$\boxed{S_{\hat{b}_1\hat{b}_2}}$

	1	2	3	4	5	6	7	8	9	10
1	0.024									
2	-0.007	0.020								
3	0.000	0.000	0.022							
4	0.000	0.000	$\boxed{0.007}$	0.026						
5	0.000	0.000	-0.020	-0.012	0.039					
6	0.000	0.000	0.000	0.000	0.000	307.609				
7	0.000	0.000	0.000	0.000	0.000	-84.379	143.000			
8	0.000	0.000	0.000	0.000	0.000	23.145	-72.102	224.610		
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.475	
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	342.192

SPSS

Syntax:

```
data list free
   / s x m1 m2 y.
begin data
    1    51     41      54     59
    2    40     34      51     60
             .
             .
end data.

*Equation 5.1.
regression
 /variables=x y m1 m2
 /dependent=y
 /enter=x.

* Equation 5.2.
regression
 /variables=x y m1 m2
 /statistics=defaults bcov
 /dependent=y
 /enter= x m1 m2.

* Equation 5.3.
regression
 /variables=x y m1
 /dependent=m1
 /enter=x.

*Equation 5.4.
regression
 /variables=x y m2
 /dependent=m2
 /enter=x.
```

SPSS Output:

Regression

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	34.727	8.925		3.891	.000
	x	.708	.173	.552	4.081	.000

a. Dependent Variable: y

Regression

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	9.123	10.478		.871	.390
	x	.112	.207	.087	.541	.592
	m1	.569	.157	.571	3.629	.001
	m2	.530	.170	.383	3.123	.004

a. Dependent Variable: y

Coefficient Correlations[a]

Model			m2	m1	x
1	Correlations	m2	1.000	.297	-.370
		m1	.297	1.000	-.689
		x	-.370	-.689	1.000
	Covariances	m2	.029	.008	-.013
		m1	.008	.025	-.022
		x	-.013	-.022	.043

a. Dependent Variable: y

Regression

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	7.097	8.129		.873	.388
	x	.840	.158	.653	5.319	.000

a. Dependent Variable: m1

Regression

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	40.711	7.514		5.418	.000
	x	.222	.146	.239	1.520	.137

a. Dependent Variable: m2