Longitudinal Mediation Analysis (Chapter 8)

Assumptions
Unique Issues with Longitudinal Relations
Two-wave Mediation Models
Three or more wave Mediation Models

1

More on Temporal Order Assumption

- Assume temporal ordering is correct: X before M before Y.
- Assume that relations among X, M, and Y are at equilibrium so the observed relations are not solely due to when they are measured, i.e., if measured 1 hour later a different model would apply.
- Assume correct timing and spacing of measures to detect effects.
- But manipulations target specific times with many patterns of change over time.

2

Mediation is a Longitudinal Model

- A mediator is a variable in a chain whereby an independent variable causes the mediating variable which in turn causes the outcome variable—these are longitudinal relations. X, M, and Y in single mediator model imply longitudinal relations even if measured at the same time.
- For a single mediator model, temporal order for X is clear when it represents random assignment, but the temporal order of M and Y must be based on prior research or theory.

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Timing of relations

- When does X affect M or M affect Y?
- Triggering, cascading, and other timing processes (Tang & DeRubeis, 1999; Howe et al., 2002)
- Tang & DeRubies (1999) found evidence that change in therapy occurs within the first few sessions.
- How are decisions made about timing? Not often considered in research projects except with respect to when a manipulation is made and the easiest time for data collection.
- Timing is crucial for deciding when to collect longitudinal measures (Collins & Graham, 2003).

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Cross-sectional mediation 1

- Gollob & Reichardt (1991) describe three limitations of cross-sectional mediation
- 1. Takes time for effects to occur-may not be enough time for X to affect M to affect Y if variables are measured at the same time.
- 2. Variables have effects on themselves-time 1 has an effect on time 2 etc.
- 3. Size of effect depends on time lag-effect 1 day apart is likely different from an effect 1 year apart.
- They specified a latent longitudinal model with prior measures as latent and lots of assumptions.
- Cross-section is a snapshot of relations

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Cross-sectional mediation 2

- Cole & Maxwell (2003) and Maxwell & Cole (2007) demonstrate limitations with cross-sectional mediation relations as described by Gollob & Reichardt (1991).
- They present reasons for differences between cross-sectional and longitudinal mediation relations. Show that many studies use cross-sectional data to assess mediation.
- Maxwell & Cole (2007) present formulas for the bias if cross-sectional rather than longitudinal data are used to assess mediation.

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Cross-sectional mediation 3

- Cross-sectional X, M, and Y. Rank order of value of X is associated with rank order of value of M which is associated with the rank order of value of Y.
- Two-wave X, M, and Y. Rank order of change in X is associated with rank order of change in M which is associated with the rank order of change in Y.
- Rank Order of the value of M is different than change in M. Relations among change in variables seem more compelling than relations among rank order of variables.

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Cross-sectional models: Summary

- Models are often cross-sectional.
- These models assume that a system has reached an equilibrium so observed relations are not just due to the particular point of observation.
- But systems may be dynamic and change over time in complicated ways.
- Meaning of cross-sectional relations (relation between rank order of level) is different from longitudinal relations (relation of rank order of change).
- Cross-sectional mediation may differ in many ways from longitudinal mediation (Cole & Maxwell, 2003; Gollob & Reichardt 1991).

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Does the study of mediation exclude cross-sectional data?

- Cross-sectional information is often used to infer relations in fields such as geography and astronomy and by detectives, physicians, and historians.
- Some cases where cross-sectional relations are more important than longitudinal change, e.g., legislator basing funding decisions based on change or level of a problem; school funding based on level or change in achievement.

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Are there variables that represent changes over time when measured once?

- Age of onset: Started regular smoking at age 15. Date of first arrest.
- Drug use last week, exercise last month.
- X measured at the first wave, M measured at the second wave, and Y measured at the third wave.
- Others?

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Benefits of Longitudinal Data

- Time-ordering of X to M to Y is investigated. Can shed light on whether changes in M precede changes in Y.
- Both cross-sectional and longitudinal relations can be examined.
- Removes some alternative explanations of effects, e.g., effects of static variables can be removed.

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What if repeated measures of X, M, and Y are available?

- Measures of X, M, and Y at two time points allow for several options, difference score, ANCOVA, residualized change score, relative change…
- Measures of X, M, and Y at three or more time points allow for many alternative longitudinal models.
- For many examples in this class, X is measured once and represents random assignment of participants to one of two groups.

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Stability, Stationarity, and Equilibrium

- Stability-the extent to which the mean of a measure is the same across time. There are different kinds of stability (Wohlwill, 1973).
- Stationarity-the extent to which relations among variables are the same across time.
- Equilibrium-the extent to which a system has stabilized so that the relations examined are the same over time.

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Models for Two Waves

- Use the difference scores for X, M, and Y in the mediation regression equations. $Y_1 - Y_2$
- Use Analysis of Covariance where the baseline value of X, M, and Y is included as a predictor of the follow-up value of X, M, and Y. $Y_2 = i + bY_1$
- Residualized Change. Predict time 2 with time 1 and use the difference between the time 2 score and predicted time 2 score as the dependent variable., $Y_2 - Y_{2\text{Predicted by Y1}}$.
- Note that difference score and residualized change score make the two-wave model into a single mediator model.

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Reliability of the Difference score

- Cronbach & Furby (1970) difference scores are unreliable because the difference is just error.
- Rogosa (1998) lack of change is the explanation of low reliability.
- Singer and Willett (2003) reliability of change is different than reliability of a measure.

	Reliability of test		
$r_{Time1,Time2}$	.7	.8	.9
.5	.4	.6	.8
.6	.25	.5	.75
.7	**.00**	.33	.67
.8		.00	.50
.9			.00

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ANCOVA vs. Residualized Change Scores

- Theoretically the residualized change score approach is similar to ANCOVA since both analyses adjust for pretest measurement. For mediation the residualized change score does not account for baseline relation between M and Y.
- The statistical adjustment that generates the residuals in the residualized change score method uses the *total regression coefficient* for Y on X (that is, the regression coefficient of Y on X across all cases, ignoring group membership), whereas ANCOVA adjustment is based on the regressions of Y on X within each group pooled across groups, the *pooled within class regression coefficient* of Y on X.
- As a result, if there is no baseline imbalance between groups, ANCOVA and residualized change scores produce similar results. However if the groups differ at baseline, then residualized change scores can lead to an underestimated treatment effect. For mediation analysis ANCOVA is better because it includes the relation between M and Y at baseline.

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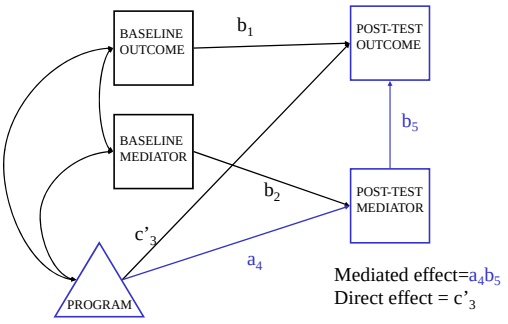
Two-wave Longitudinal Model



Mediated effect=a_4b_5
Direct effect = c'_3

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Summary of Two-Wave Models

- Difference score versus ANCOVA models. Randomized X then ANCOVA is best. But there are other measures. If there is a difference in the results between the two models, check for baseline differences.
- Difference score and residualized change measures are useful because they transform two measures to one measure, i.e., the difference score combines the time 1 mediator and time 2 mediator so all the models that we have discussed in this course so far can be applied.
- Meaning of mediation with the different models differ: Correlated change scores, correlated adjusted time 2 scores. Note issue of Lord's paradox for the M to Y relation because M is not randomized.
- ANCOVA is generally the best approach because it models all the information from two waves of data.
- Models with two waves are half-longitudinal because some relations are cross-sectional but Cole and Maxwell suggest using a from X1 to M2 and b from M1 to Y2.
- More options with more waves of data. More complexity too though.

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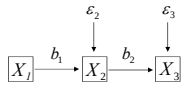
Models for Three or More Waves

- Autoregressive Models
- Latent Growth Curve Models (LGM)
- Latent Change Score Models (LCS)
- Autoregressive and Latent Growth Curve Models (ALT)
- Differential Equation Models (DEM)

Autoregressive (Jöreskog, 1974)

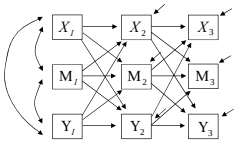


$$X_2 = b_1 X_1 + \varepsilon_2$$

$$X_3 = b_2 X_2 + \varepsilon_3$$

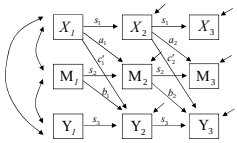
$$\sigma^2_{x1}, \sigma^2_{\varepsilon 2}, \sigma^2_{\varepsilon 3}, b_1, b_2$$

General Autoregressive Model



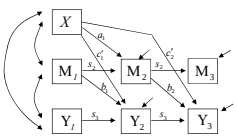
Note: All residuals are correlated

Autoregressive Model with Time-Ordered Mediation, Cole & Maxwell (2003)



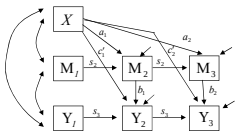
Note: All residuals are correlated

Autoregressive Model with Time-Ordered Mediation (MacKinnon 1994, 2008)



Note: Residuals at the same time are correlated

Autoregressive Model with Contemporaneous Effects for M to Y (MacKinnon 2008; Marsh 1993)



Note: Residuals at the same time are correlated

Autoregressive Models

- Many mediated effects. Standard error of the sum of (or any function) the indirect effects can be derived with the multivariate delta method, e.g., for the Cole and Maxwell (2003) overall indirect effect standard error on page 564.
- Model does not allow for random effects for individual change and does not include modeling of means. Change in growth of means is an important aspect of longitudinal data.

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Latent Growth Model (LGM)

- LGM model change over time by estimating an intercept and slope for change in variables. These models can be used to investigate mediation by estimating change over time for the mediator and change over time for the outcome. The relation between the change in the mediator and change in the outcome is represented by the *b* path (Cheong et al. 2003).
- The causal direction of correlated change is ambiguous. Another LGM estimates change in the mediator at earlier time points and relates to change in the outcome at later time points providing more evidence for temporal precedence of the mediator.

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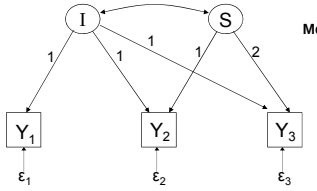
Latent Growth Model (LGM)



Meredith & Tisak (1990)

$$Y_1 = I + 0S + \varepsilon_1$$
$$Y_2 = I + 1S + \varepsilon_2$$
$$Y_3 = I + 2S + \varepsilon_3$$
$$\sigma^2_{\varepsilon_1}, \sigma^2_{\varepsilon_2}, \sigma^2_{\varepsilon_3}, \sigma^2_I, \sigma^2_S, \rho_{IS}, Means$$

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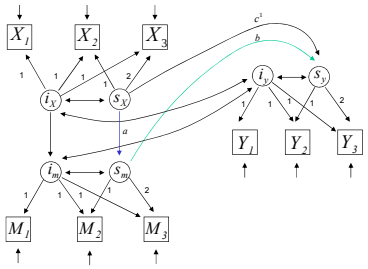
Latent Growth Curve Mediation Model



Cheong et al., 2003

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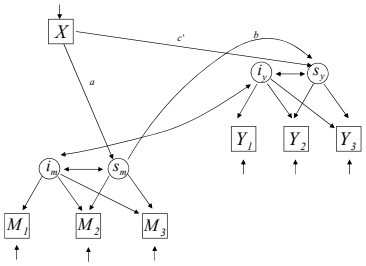
Latent Growth Curve



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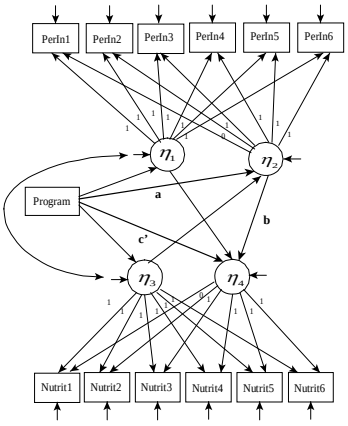

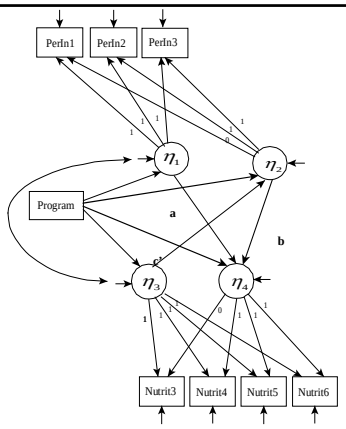

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Latent Change Score Models

- LCS parameterize models by fixing parameters so that change between adjacent waves is analyzed.
- Really a special case of latent growth curve modeling but with growth between adjacent waves.
- More complicated change over time can be made by picking different coefficients and second order factors.
- Promising model not often used for mediation analysis.

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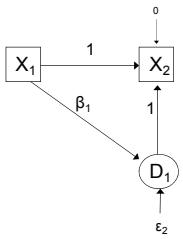
Latent Change Score (LCS)Two Waves McArdle (2001)

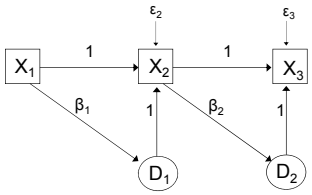


$$X_2 = (1)X_1 + (1)D_1$$
$$X_2 - X_1 = (1)D_1$$
$$X_2 - X_1 = D_1$$
$$D_1 = B_1 X_1 + \varepsilon_2$$

$$\sigma^2_{x1}, \sigma^2_{\varepsilon 2}, \beta_1$$

9/24/2016 D.P. MacKinnon

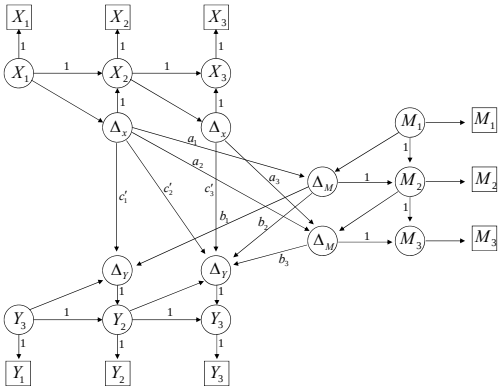
Latent Change Score (LCS) McArdle (2001)



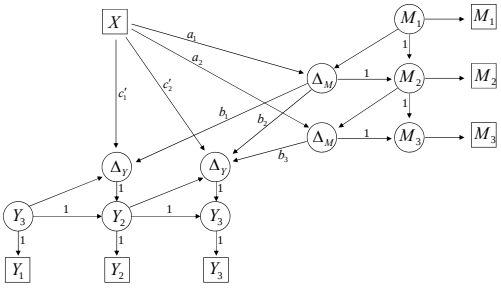
$$X_2 = X_1 + D_1 + \varepsilon_2$$
$$X_3 = X_2 + D_2 + \varepsilon_3$$

$$D_1 = B_1 X_1$$
$$D_2 = B_2 X_2$$
$$\sigma^2_{x1}, \sigma^2_{\varepsilon 2}, \sigma^2_{\varepsilon 3}, \beta_1, \beta_2, Means$$

Latent Change Score Mediation Model



Latent Change Score Mediation Model Single X



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Longitudinal models for a steroid prevention project (ATLAS)

- Adolescents Teaching and Learning to Avoid Steroids (ATLAS). Randomized high school football teams in Oregon and Washington to receive the steroid prevention program or an information only group. Just individual data here.
- Measured the same persons over repeated occasions. Here we will look at four repeated measures. The dependent variable is intentions to use steroids.
- Linn Goldberg (OHSU) principal investigator. For more on the program see Goldberg et al. (1996) and for mediation see MacKinnon et al., (2001).
- Program delivered after baseline measurement. In general, timing of the mediators should be relatively quick for knowledge and beliefs measures. It may take longer for norms measures. Four waves of measurement for the models studied.

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Analysis decisions

- LGM model, slope coded as 0 1 * 1 where * indicates a free parameter. Note that there was a booster after the 3rd measurement. If the model was not identified, then loadings were 0 2.5 * 14.5 to represent the months from baseline. All LGM models had RMSEA lower than .041 (lowest .019).
- Autoregressive model. Tested for stationarity in the *a* and *b* paths. Stationarity observed more often for *b* paths and less often for *a* paths, as expected. All RMSEAs lower than .088 (lowest was .068).

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LGM and Autoregressive mediation effects

Mediator	LGM	Autoregressive	
	ab(se) z	*a1b2(se) z*	*a2b3(se) z*
Knowledge	-.28(.12) -4.88	-.08(.02) -4.90	-.03(.04) -0.48
Coach Tol	-.11(.05) -2.27	-.02(.01) -3.24	-.00(.00) -0.37
Team as info	-.21(.06) -3.42	-.04(.02) -3.30	.01(.01) 0.78
Peer as info	-.12(.05) -2.43	-.04(.01)-2.30	-.01(.00) -1.61
Reasons not use	-.12(.04) -2.98	-.02(.01) -3.01	.00(.00) 0.61
Normative bel	-.12(.07) -1.64	-.00(.00) -0.14	-.01(.01) -0.98

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Measurement

- Does the measure have the same meaning at each wave? So it is possible that the system is stationary and stable but the measurement of the construct changes.
- Multiple indicator latent variable models are ideal.
- Important to consider measurement of constructs at each wave and measurement of change over time separately.

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X, M, and Y may differ over time

- X, M, and Y at an earlier developmental stage may differ from X, M, and Y at a later stage. For example social norms may be important mediators of drug use prevention in middle school but positive alcohol expectancies may be important mediators for programs targeted at the transition from high school to college. Onset may be important for 13 year olds and heavy use may be important for 21 year olds. Intervention to change expectancies for 13 year olds may differ from expectancy interventions for 21 year olds.
- Many manipulations have an initial program and booster sessions so that even X differs over time, e.g., adaptive interventions.

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Transitions as Critical Periods

- Transitions are important, e.g., home to elementary school, elementary to high school, high school to workforce/college. There are many aspects to these transitions including environmental, biological, social, and family changes.
- For example, interventions to reduce aggressive behavior from home to elementary school may focus on improving educational achievement while interventions to reduce aggressive behavior for the transition from middle to high school may focus conflict resolution and self-control.

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Types of change over time

- Change in X, M, and Y and also relations between change in X on M and change in M on Y.
- Cumulative: There may be cumulative effects such that more M yields more Y.
- Threshold: Once a mediator gets to a certain level, then it will change Y.
- Cascading: Once a proximal mediator changes it changes a more distal mediator and finally an outcome variable.
- Phase shift: Once a level of a mediator is reached, the individual changes to a new level, e.g., learning a concept in algebra.
- The types of changes may differ over time.

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Type of change may differ for X to M and M to Y

- Both the X to M and M to Y relations may be the same, e.g., linear cumulative change for X to M and M to Y. Often linear change is assumed for both.
- Effects of X on M may differ from M on Y. A cumulative change in the mediator may trigger change in Y.
- The change in X to M may lead to a phase shift or new stage which then leads to a stage shift in M to Y.
- Many different possibilities requiring detailed modeling both to describe these relations and then confirmatory models for the two parts of the mediation relation.

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Sleeper Mediation Effects

- Effects on mediators may have beneficial effects later. For example an intervention to increase calcium consumption among teenage women may not yield beneficial effects on osteoporosis until much later.
- Interventions to improve educational achievement in elementary school may reduce problems in young adulthood.
- Social competence skills learned in elementary school may reduce violence as adults.
- Norm change to prevent gateway drug use may reduce heavy use later.

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Correspondence between Measurement and Population Change

- Match between theoretical population model and timing of measures is crucial.
- Many waves of data collection do not ensure correct longitudinal modeling.

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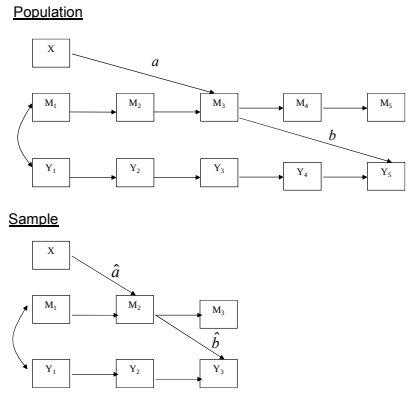

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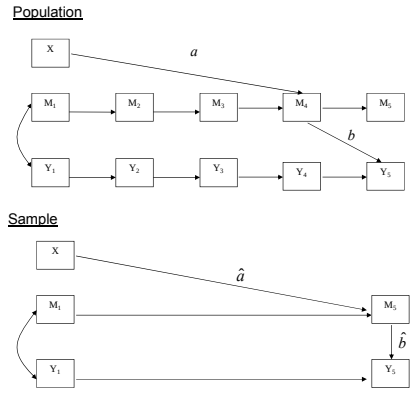

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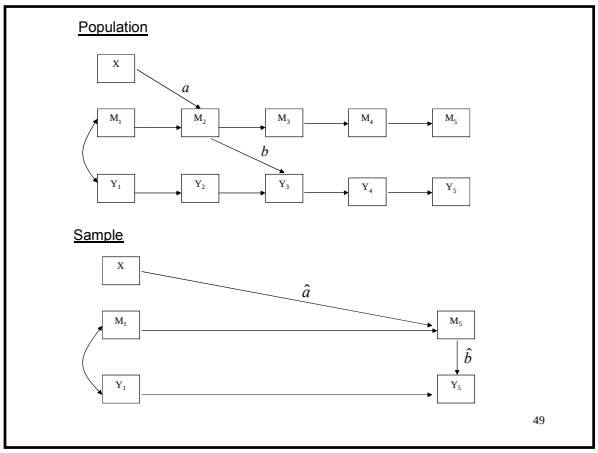

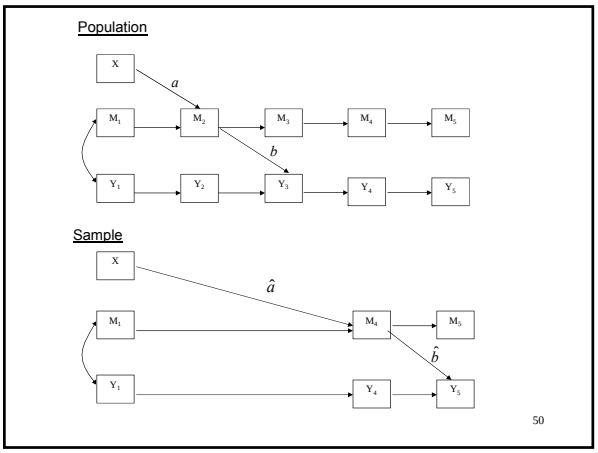

Summary

- Longitudinal data provide more information.
- Many alternative models that provide different information about mediation effects.
- Often requires an iterative process to model longitudinal data.
- Perhaps estimate all models on the same data and compare results. So far different models lead to comparable conclusions.
- Need examples of applying the models to real data.

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