Measures of Effect Size (Chapter 4)

- There are several measures of effect size for the mediation model
  - Effect size measures for individual paths
  - Effect size measures for the mediated effect

1

Measures of Effect Size for Paths

- Correlation between X and M for the $\hat{a}$ coefficient.
- Partial correlations for $\hat{b}$ and $\hat{c}'$. Correlation of .1, .3, and .5 correspond to small, medium, and large effects (Cohen 1988)
- Standardized betas for $\hat{b}$, $\hat{c}'$, and $\hat{a}$. Change in standard deviations in the dependent variable for a standard deviation change in the independent variable

2

Correlation Measures of Effect Size

- Correlation between X and M ($\hat{a}$) $\quad r_{XM}$
- Correlation between M and Y partialled for X ($\hat{b}$) $\quad r_{YM.X} = \dfrac{r_{MY} - r_{XY} r_{XM}}{\sqrt{(1-r_{XY}^2)(1-r_{XM}^2)}}$
- Correlation between X and Y partialled for M ($\hat{c}'$) $\quad r_{YX.M} = \dfrac{r_{XY} - r_{MY} r_{XM}}{\sqrt{(1-r_{MY}^2)(1-r_{XM}^2)}}$

3

Standardized Beta Measures of Effect Size

- Standardized Beta between X and M ($\hat{a}$) $\quad \hat{a}_s = r_{XM}$
- Standardized Beta between M and Y adjusted for X ($\hat{b}$) $\quad \hat{b}_s = \dfrac{r_{MY} - r_{XM} r_{YX}}{1-r_{XM}^2}$
- Standardized Beta between X and Y adjusted for M ($\hat{c}'$) $\quad \hat{c}'_s = \dfrac{r_{XY} - r_{XM} r_{YM}}{1-r_{XM}^2}$

4

Effect size for the water consumption study

- Correlation and partial correlation effect size measures were: $\hat{a}$ = .371, $\hat{b}$ = .411, $\hat{c}'$ = .222, $\hat{c}$ = .361
- Standardized betas were: $\hat{a}$ = .371, $\hat{b}$ = .413, $\hat{c}'$ = .208, $\hat{c}$ = .361

5

Effect size for Word Experiment Data

- Correlation and partial correlation effect size measures were: $\hat{a}$ = .623, $\hat{b}$ = .390, $\hat{c}'$ = .040, $\hat{c}$ = .337
- Standardized betas were: $\hat{a}$ = .623, $\hat{b}$ = .470, $\hat{c}'$ = .044, $\hat{c}$ = .337

6

Measures of Mediated Effect Size

- Proportion mediated: $\frac{\hat{a}\hat{b}}{\hat{c}} = \frac{\hat{a}\hat{b}}{\hat{a}\hat{b}+\hat{c}'} = 1-\left(\frac{\hat{c}'}{\hat{c}}\right)$

(Estimators are equal for OLS regression but not for logistic and probit regression)

- Ratio of mediated to direct effect: $\frac{\hat{a}\hat{b}}{\hat{c}'}$
- R-squared attributable to the mediated effect:

$$r^2_{YM} - (R^2_{Y,MX} - r^2_{YX})$$

7

Mediated effect size in the Water Consumption study

- Proportion mediated was $\frac{\hat{a}\hat{b}}{\hat{c}}$ =.1527/.3604 = .4238
  - 42% of the total effect of X on Y was through the mediator M.
- Ratio of indirect to direct effect was $\frac{\hat{a}\hat{b}}{\hat{c}'}$ = .1527/.2076=.7354
  - The mediated effect was .74 the size of the direct effect controlling for the mediator.
- R^2 attributable to the mediated effect was R^2_{med} = (.2399-(.2772-.1304)) = .0931

8

Mediated effect size in the Word Experiment Data

- Proportion mediated was $\frac{\hat{a}\hat{b}}{\hat{c}}$ =2.185/2.517 =.868
- Ratio of indirect to direct effect was $\frac{\hat{a}\hat{b}}{\hat{c}'}$ = 2.185/.332=6.58
- R^2 attributable to the mediated effect was R^2_{med} = (.2476-(.2488-.1137)) = .1125

9

Other Effect Size Measures: Water Consumption Example

- Mediated effect in terms of standard deviations of the dependent variable, Y (MacKinnon, 2008).

$$Standardized_{\hat{a}\hat{b}} = \frac{\hat{a}\hat{b}}{s_y}$$

- Water consumption value was .1343 = (.1527/1.134)
  - For a one-unit increase in X, Y increases by .13 standard deviations due to mediation.
- Surrogate endpoint $\frac{\hat{c}}{\hat{a}}$ and correlation between M and Y.
  - The ideal surrogate $\frac{\hat{c}}{\hat{a}}$ = 1 and r_{MY} equals 1.
- Water consumption data $\frac{\hat{c}}{\hat{a}}$ = .613= (.2076/.3386) and r_{MY} equals .489

10

Other Effect Size Measures: Word Experiment Data

- Mediated effect in terms of standard deviations of the dependent variable, Y (MacKinnon, 2008).

$$Standardized_{\hat{a}\hat{b}} = \frac{\hat{a}\hat{b}}{s_y}$$

- Word class experiment value was .5812=(2.185/3.7594)
- Surrogate endpoint $\frac{\hat{c}}{\hat{a}}$ and correlation between M and Y.
  - The ideal surrogate $\frac{\hat{c}}{\hat{a}}$ = 1 and r_{MY} equals 1.
- Water consumption data $\frac{\hat{c}}{\hat{a}}$ = .7073= (2.5167/3.5583) and r_{MY} equals .4976

11

Additional Effect Size Measures

Mediated effect standardized by standard deviation of both X and Y (Alwin & Hauser, 1975; Cheung, 2009). $\hat{a}\hat{b}\frac{s_X}{s_Y}$

k^2 (Preacher & Kelly, 2011, *Psychological Methods*) Proportion of the maximum possible indirect effect. Divide the observed mediated effect by the largest possible value of *ab* that could be obtained given the data. The largest possible mediated effect is a function of the observed variances and covariances among X, M, and Y. Problems with k^2 owing to nonmonotonicity shown by Wen & Fang, (2015: *Psychological Methods).*

12

Additional Effect Size Measures Water Consumption Data

ab standardized by standard deviation of X and Y $\hat{a}\hat{b}\frac{s_X}{s_Y}$

Water Consumption Data = $\hat{a}\hat{b}\frac{s_X}{s_Y} = .1527\frac{1.137}{1.135} = .153$

For a one standard deviation increase in X, Y increases by .15 standard deviations due to the mediated effect.

k^2 for Water Consumption Data = .153/.992=.154

The observed proportion of the maximum possible indirect effect is .15.

13

Additional Effect Size Measures Word Experiment Data

Mediated effect standardized by standard deviation of X and Y

Word Spring 2012 Data = $\hat{a}\hat{b}\frac{s_X}{s_Y} = 2.185\frac{.5037}{3.7594} = .2928$

k^2 for Word Data =2.185/8.843=.2471

Proportion of the maximum possible indirect effect.

14

Standardized Effect Size Measures

- Mediated effect in terms of the change in standard deviation units of Y for a one unit change in X. Useful for binary X or when one unit change is desired. (Mplus STDY)

$$\frac{\hat{a}\,\hat{b}}{s_Y}$$

- Mediated effect in terms of the change in standard deviation units of Y for a one standard deviation change in X. Useful for continuous X. (Mplus STDXY)

$$\hat{a}\hat{b}\frac{s_X}{s_Y}$$

15

Simulation Results

- Mackinnon, Warsi, & Dwyer (1995), Marcia Taborga's masters thesis, MacKinnon, Fairchild, Yoon, & Ryu (2007), and Fairchild, MacKinnon,Taborga, & Taylor (2009).
- Correlation and standardized beta values for individual paths work well at reasonable sample sizes of 50 etc.
- Ratio requires at least N of 1000. Proportion requires sample size of 500 unless effect sizes are large then OK for as small as 100. Standardized mediated effect and mediation R-squared seem to work reasonably well and show promise at small sample sizes.
- More work needs to be done but at this point standardized effect sizes are recommended.

16

Simulation Results (continued)

- Some work evaluating the bias and stability from sample to sample of ab/s_Y, $ab(s_X)/s_Y$, k^2, the proportion, and ratio mediated showed that ab/s_Y, $ab(s_X)/s_Y$, and k^2 have lower relative bias and more stability than the proportion and ratio mediated even at sample sizes as low as N=10 (Miočević, O'Rourke, & MacKinnon, 2014).
- You can report more than one effect size for a given study; some effect sizes have more intuitive interpretations for your data.

17

Summary

Effect sizes for individual paths in the mediated effect and also the mediated effect.

Use correlation and standardized betas for individual paths.

Standardized mediated effect measures are reasonable either for a one unit change in X or a standard deviation change in X. The proportion mediated is widely used but may not be stable at smaller sample sizes.

Can derive standard errors for any function using the multivariate delta method. Could also use the bootstrap to find confidence intervals and Bayesian estimation to find the credibility intervals. Can do this with the Mplus MODEL CONSTRAINT command.

18

When a third variable increases or reverses the relation between X and Y.

- In most situations, the relation between X and Y is reduced when the third-variable is included because it is a mediator or a confounder and it explains part of the relation of X and Y. There are cases where the X to Y relation gets bigger or reverses sign when a third variable is included.
- A suppressor variable is a variable that increases the magnitude of the relation between X and Y when it is included in the analysis.
- A distorter variable changes an X to Y relation such that when it is included, a relation emerges or changes in sign.
- A suppressor or distorter could be a mediator or confounder.
- A covariate is not a suppressor or distorter because it does not change the relation between X and Y.

19

Suppressor Example

- Horst (1941) evaluated the relation between mechanical ability and pilot performance. The relation increased when verbal ability was included.
- Mechanical ability and pilot performance are strongly related. It takes verbal ability to complete the mechanical ability test. So removing verbal ability from the test, yields a more accurate (and larger) estimate of mechanical ability and pilot performance.
- So magnitude of the relation between mechanical ability and pilot performance increased when verbal ability was included. It is a confounder not a mediator because it doesn't really make sense that mechanical ability causes verbal ability which causes pilot performance.

20

Distorter Example 1

A distorter third-variable reverses the sign of the relation between X and Y or changes a zero relation between X and Y to a nonzero relation.

Positive relation between suicide rate and marital status overall. More likely to commit suicide if married seems unusual. When age is included in these analyses, there is a negative relation between marriage and suicide rate for each age (Rosenberg, 1968, p. 84). Age is a confounder not a mediator because age does not make sense as a mediator between marriage and suicide.

21

Distorter Example 2

A distorter variable exhibits what has been called the Simpson's paradox, also known as the reversal paradox (also Lord's paradox). These effects occur when the overall relation between two variables differs from the relation across levels of the confounding variable.

Two Treatments for kidney stones: Treatment A was best both for small 93% vs 87% success and large 73% vs 69% kidney stones but Treatment B was better if size of stone was not considered 78% versus 83% success. The overall relation of treatment to success differs from the adjusted effect because of different sample sizes in each group.

22

Distorter

"one may be equally misled in assuming that an absence of relation between two variables is real, whereas it may be due .. to the intrusion of a third variable" (Rosenberg, 1968, p. 84).

23

Inconsistent Mediation Models

Inconsistent mediation models occur when the relation of X to Y increases in magnitude when the mediator is included in the analysis (see MacKinnon, Krull, & Lockwood 2000).

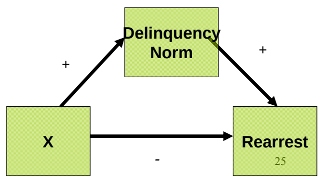
There is a mediation relation because the mediator transmits the effect of the independent variable to the dependent variable. Inconsistent mediation can occur whether or not $\hat{c}$ is statistically significant. The only requirement is that $\hat{c}$' is larger in magnitude than $\hat{c}$.

Are inconsistent mediation effects rare?

24

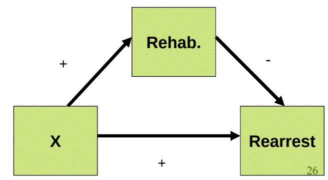
Inconsistent Mediation Example: Delinquency

- Program to reduce juvenile delinquency brings high risk persons together for a special program. But the program increases the social norm that juvenile delinquency is common and that social norm increases subsequent delinquency. But overall, the program reduces juvenile delinquency.



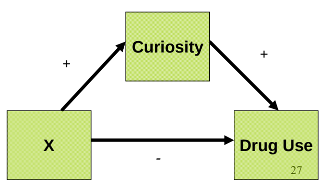
Inconsistent Mediation Example: Incarceration

Incarceration increases rehabilitation and rehabilitation reduces rearrest. But overall, incarceration increases rearrest because of exposure to pro-crime norm, for example.



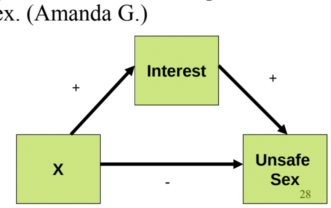
Inconsistent Mediation Example: Drug Prevention

- Drug prevention increases curiosity about drugs. But overall, prevention reduces drug use behavior. (Matt)



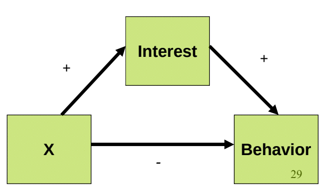
Inconsistent Mediation Example: STD Prevention

- Condom promotion increases interest in sex which increases interest in sex (A criticism of safe sex interventions). But overall condom promotion reduces unsafe sex. (Amanda G.)



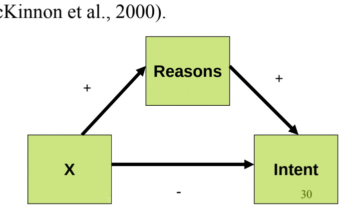
Inconsistent Mediation Example: Obesity Prevention

- Obesity prevention increases interest in food which increases overeating. But overall the program reduces overeating. (Angela)

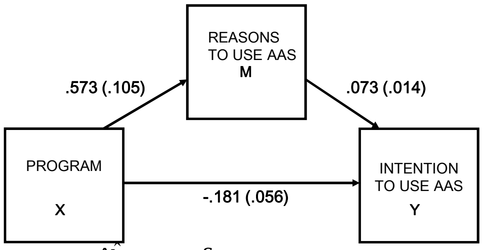


Inconsistent Mediation Example: Steroid Prevention

- Steroid prevention program increases reasons to use steroids and reasons to use steroids increases intentions to use steroids. But overall the intervention reduces intentions to use steroids (MacKinnon et al., 2000).

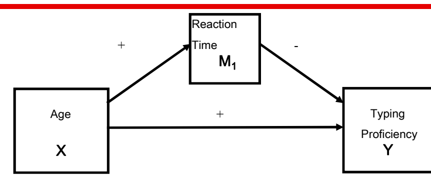


Inconsistent mediation in ATLAS Data

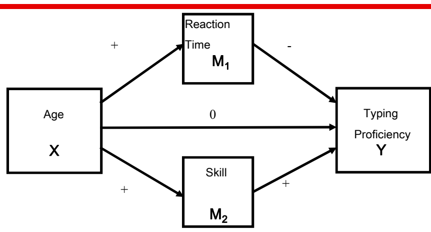


Mediated effect $\hat{a}\hat{b}$ = .042 ($s_{\hat{a}\hat{b}}$ =.011)
Direct effect $\hat{c}'$ = -.181 ($s_{\hat{c}'}$=.056);Total effect = $\hat{c}$= -.139 $s_{\hat{c}}$ =.056

Mediators of age on typing (Salthouse, 1984)



Multiple Mediator Model Preview: Opposing mediators for the null effect of age on typing (Salthouse, 1984; Baltes & Baltes,1990)



Inconsistent Mediation Models Summary

Are inconsistent mediation effects rare?

Are there types of inconsistent mediation relations?
Interest, norm, opposing mediation effects…

More on inconsistent mediation in multiple mediator models. An inconsistent mediation model has at least one mediated effect that has a different sign than the direct effect or other mediated effects.