Single Mediator Model So Far

Three Regression Equations
Estimates of the mediated effect, significance testing and confidence limits
Simulation study results for significance testing and confidence limit estimation
Reasons for discrepancies among tests
Mediator and Confounder Revisited
Inconsistent Mediation Revisited
Effect Size

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Three Major Types of Single Sample Tests for the Mediation Effect

(1) Causal Steps: Series of tests described in Baron & Kenny (1986) and Judd & Kenny, (1981).

(2) Difference in Coefficients: $\hat{c}-\hat{c}$ estimator, e.g., from Clogg et al. (1992)

(3) Product of Coefficients: $\hat{a}\hat{b}$ estimator, e.g., from Sobel (1982)

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Three Mediation Equations

$Y = i_1 + c\,X + e_1$

$Y = i_2 + c'\,X + b\,M + c_2$

$Y = i_3 + a\,X + e_3$

With XM interaction
$Y = i_4 + c'\,X + b\,M + h\,XM + e_4$

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Significance Testing and Confidence Limits

Recommend product of coefficients estimation of the mediated effect and standard error. Recommend joint significance, distribution of the product, and bootstrap for confidence limit estimation and significance testing. Bias-corrected bootstrap has the most power but can have slightly higher Type I error rates that occur in rare circumstances.

Note that now the distribution of the product test is only available for two-path mediated effects. Joint significance and resampling methods work for any model even complicated ones.

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Reasons for Differences Among Methods

Requirement for significant total effect, $\hat{c}$, and requirement that $\hat{c}'$ is nonsignificant reduces statistical power of BK and JK causal steps methods.

Assumption that the mediated effect divided by its standard error has a normal distribution is incorrect.

Mediation is fundamentally a test of two paths corresponding to *a* and *b* paths.

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What is the problem with requiring $\hat{c}$ to be statistically significant? #1

Can drastically reduce power to detect a mediation effect and power is reduced as mediation approaches complete mediation. Ironic that use of this criteria leads to lowest power for complete mediation models when complete mediation is the most defensible mediation conclusion from a research study.

Subgroups of persons who have opposing mediated effects, e.g. mediation relation for males is opposite of that for females so $\hat{c}$ is nonsignificant when sex is ignored.

Test of $\hat{c}$, is a statistical test that can be wrong (Type 1 and 2 Errors). Because the null hypothesis of $c = 0$ is not rejected does not mean that it should be accepted that $c = 0$ (same as any null hypothesis).

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What is the problem with requiring $\hat{c}$ to be statistically significant? #2

- Test of $\hat{a}\hat{b}$ is more powerful than test of $\hat{c}$, i.e., mediation more precisely explains how X affects Y.
- Lack of statistically significant $\hat{c}$ is very important for mediation analysis because failure of action, conceptual, or both theories is critical for future studies.
- Inconsistent mediation relations are possible because adding a mediator may reveal a mediation relation.
- Note the test of $\hat{c}$ is important in its own right but is a different test than the test for mediation.

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When the test of Mediation has more power that the test of the Total Effect?

- The test of $\hat{a}\hat{b}$ has more power than the test of $\hat{c}$ when effects are small and sample size is large, and when effects are large and sample size is small.
- When $\hat{a}\hat{b}$ is equal to $\hat{c}$, the test of $\hat{a}\hat{b}$ is always more powerful than the test of $\hat{c}$.
- This occurs because the standard error of $\hat{c}$ is larger than the standard error of $\hat{a}\hat{b}$.

O'Rourke, H. P., & MacKinnon, D.P. (2015). When the test of mediation has more power that the test of the total effect. *Behavior Research Methods, 47*, 424-442.

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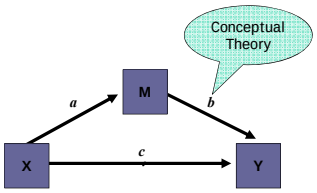
Breaking Down the Mediated Effect: Conceptual Theory Failure

- Conceptual theory outlines how hypothesized mediators are linked to outcomes of interest.
  - Are these the right mediators? Are they causally related to the dependent variable?



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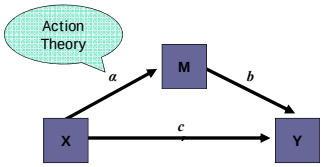
Breaking Down the Mediated Effect: Action Theory Failure

- Action theory outlines how a manipulation, X, relates to hypothesized mediators
  - Can these mediators be changed? How do we change these mediators?



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Mediator, Confounder, Moderator, Covariate

- Mediator-a variable that is intermediate in a causal sequence such that X causes the mediator and the mediator causes Y. The relation between X and Y changes when adjusted for the mediator.
- Confounder-a variable that is related to both X and Y but is not in a causal mediation sequence. The relation between X and Y changes when adjusted for the confounder.
- Covariate- a variable that is related to X or Y or both. The relation between X and Y does not appreciably change when adjusted for the covariate.
- Moderator-a variable where the relation of X to Y is different at different values of the moderator.

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When a third variable increases the relation between X and Y.

In most situations, the relation between X and Y is reduced when the third-variable is included because it is a mediator or a confounder and it explains part of the relation of X and Y. There are cases where the X to Y relation gets bigger or reverses sign when a third variable is included.

A suppressor variable is a variable that increases the magnitude of the relation between X and Y when it is included in the analysis.

A distorter variable changes an X to Y relation such that when it is included, a relation emerges or changes in sign.

A suppressor or distorter could be a mediator or confounder.

A covariate is not a suppressor or distorter because it does not change the relation between X and Y.

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Inconsistent Mediation Models

Inconsistent mediation models occur when the relation of X to Y increases in magnitude when the mediator is included in the analysis (see MacKinnon, Krull, & Lockwood 2000).

There is a mediation because the mediator transmits the effect of the independent variable to the dependent variable. Inconsistent mediation can occur whether or not $\hat{c}$ is statistically significant. The only requirement is that $\hat{c}'$ is larger in magnitude than $\hat{c}$.

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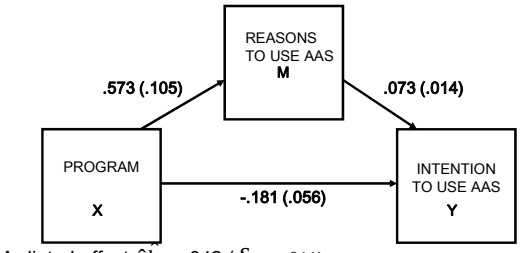
Inconsistent mediation in ATLAS Data



Mediated effect $\hat{a}\hat{b}$ = .042 ($s_{\hat{a}\hat{b}}$ =.011)

Direct effect $\hat{c}'$ = -.181 ($s_{\hat{c}'}$ =.056);Total effect = $\hat{c}$= -.139 $s_{\hat{c}}$ =.056

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Inconsistent Mediation Models

Are inconsistent mediation effects rare?

More on inconsistent mediation in multiple mediator models. An inconsistent mediation model has at least one mediated effect that has a different sign than the direct effect or other mediated effects.

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Effect Size

Effect sizes for individual paths in the mediated effect: correlation and standardized regression coefficients.

Effect sizes for the mediated effect: standardized mediated effect, proportion mediated, R^2 mediated, proportion of total possible mediated effect.

Can obtain confidence intervals and tests of significance by deriving the standard error of any function of random variables with the multivariate delta method. Can also use the bootstrap to obtain confidence intervals.

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Summary

Even the single mediator model is complex.
Regression coefficients are used to obtain estimates of the different effects in the mediation model.
Significance testing and confidence limit estimation complicated by the non-normal distribution of the product.
Consistent and Inconsistent mediation models.
Product of coefficient methods extend to more complicated models.
Some methods and statistics will no longer be appropriate for more complicated models.
More complicated mediation models primarily address violations of assumptions of the single mediator model, such as omitted variable bias, temporal precedence, measurement error, moderation and mediation, categorical data, multilevel data….

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