Parallel, Sequential, and SEM Mediator Models (Chapters 5 and 6)

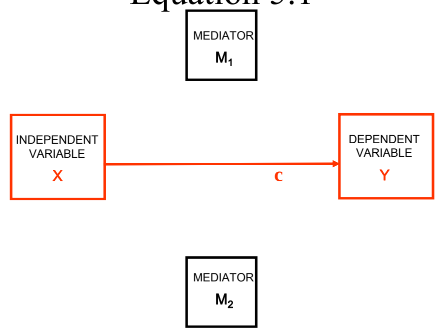
More detail on the two mediator model: a parallel two mediator model and a sequential two mediator model.

General SEM model for mediation.

Mplus demonstrations because it has capabilities for estimating specific mediated effects.

1

Equation 5.1

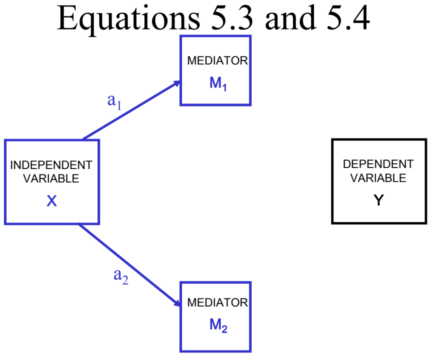


1. The independent variable is related to the dependent variable:

$\hat{Y} = \hat{c}X + \varepsilon_1$

2

Equations 5.3 and 5.4

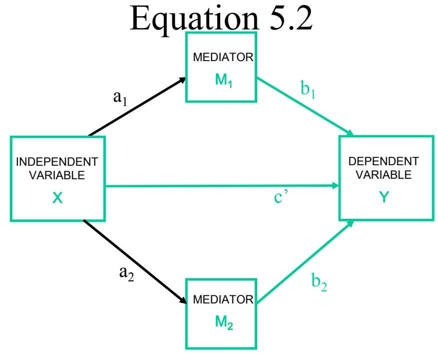


2. The independent variable is related to the potential mediators:

$\hat{M}_1 = \hat{a}_1X + \varepsilon_2, \hat{M}_2 = \hat{a}_2X + \varepsilon_3,$

3

Equation 5.2



3. The mediators are related to the dependent variable controlling for exposure to the independent variable:

$\hat{Y} = \hat{c}'X + \hat{b}_1M_1 + \hat{b}_2M_2 + \varepsilon_6$

4

Mediation Effects

Mediated effects = a_1b_1, a_2b_2

Standard error = $\sqrt{a_i^2 s_{b_i}^2 + b_i^2 s_{a_i}^2}$

Total mediated effect= $a_1b_1 + a_2b_2 = c - c'$

Direct effect= c' Total effect= $a_1b_1 + a_2b_2 + c' = c$

Test for significant mediation:

$z' = \dfrac{a_1b_1}{\sqrt{a_i^2 s_{b_i}^2 + b_i^2 s_{a_i}^2}}$ Compare to empirical distribution of the mediated effect

5

Mplus for the Two Mediator Model

```
DATA: File is 'E:\Med Course
  2\Chap6_twomed.txtmultmedp2.txt';
  Variable:
  Names are X M1 M2 Y;
  Usevariables are y M1 M2 x;
  Analysis:
  Model:
  Y on M1 M2 X; Equation 5.2
  M1 on X; Equation 5.3
  M2 on X; Equation 5.4
  M1 with M2; Covariance between the mediators
  X; Variance of X
  Model Indirect:
  Y ind X;
  Output:sampstat cinterval tech1 tech3;
```

6

MODEL INDIRECT : Y IND X;

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

	Estimates	S.E.	Est./S.E.	
Effects from X to Y				
Total	0.708	0.169	4.187	**Total effect of X on Y.**
Total indirect	0.596	0.170	3.499	**Total of both mediated effects.**
Specific indirect				
Y				
M1				
X	0.478	0.153	3.132	**Specific Mediated Effect through M1.**
Y				
M2				
X	0.118	0.083	1.409	**Specific Mediated Effect through M2.**
Direct				
Y				
X	0.112	0.197	0.570	**Direct Effect.**

Two Mediator Equations

$$\hat{M}_1 = \hat{a}_1 X + e_1$$

$$\hat{M}_2 = \hat{a}_2 X + e_2$$

$$\hat{Y} = \hat{c}' X + \hat{b}_1 M_1 + \hat{b}_2 M_2 + e_3$$

Two Mediator Matrix Equation

$$\begin{bmatrix} M_1 \\ M_2 \\ Y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ b_1 & b_2 & 0 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ Y \end{bmatrix} + \begin{bmatrix} a_1 \\ a_2 \\ c' \end{bmatrix} \mathrm{X} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Mplus Tech1 and Tech3

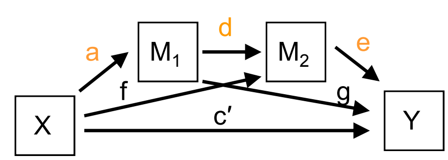
- The covariance matrix among estimates in a model is useful to obtain standard errors of functions of coefficients such as the proportion mediated, ratio of mediated to direct effect and contrasts among mediator.
- Mplus will print out the covariance among the estimates and the correlation among the estimates with the Tech3 command on the Output line.
- To be exactly sure how the Tech3 matrices are set up, use the Tech1 command which gives the order of the variables in the Tech3 matrices.

PHLAME Two Mediator Model

- Firefighter study looked at mediators of the relation between exposure to the TEAM (X) program and participants' BMI (Y) at posttest.
- Two mediators were hypothesized to be dietary support (M_1) and fruit & vegetable intake (M_2)

$X \rightarrow M_1 \rightarrow M_2 \rightarrow Y$ =
TEAM→ Dietary Support→ FV intake→ BMI

Two Mediators in a Sequence

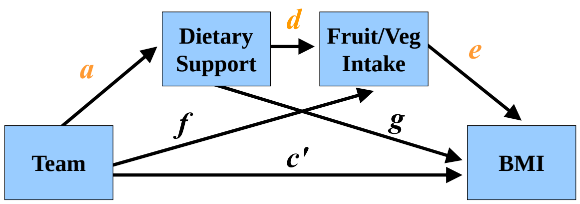


$\hat{M}_1 = a_0 + aX$ $\hat{M}_2 = d_0 + dM_1 + fX$

$\hat{Y} = e_0 + eM_2 + gM_1 + c'X$

The mediated effect is *ade*.

Two Mediators in a Sequence: The PHLAME Example



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Two Mediators in a Sequence: MPLUS Syntax for PHLAME Example

```
title:     Sequential Multiple Mediator Example;
data:      file is newmed.txt;
variable:  names are dtst wght1 bmi1 whrat1… dietsup3 team mi;
           usevariables are bmi2 total2 dietsup2 team;
           missing are all (-99.000);
analysis:  type = general missing h1;
           bootstrap=1000;
model:     bmi2 on total2 dietsup2 team;
           total2 on dietsup2 team;
           dietsup2 on team;
model indirect: bmi2 IND total2 dietsup2 team;
                total2 IND dietsup2 team;
output:    tech1 sampstat;
```

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MPLUS Model Indirect Statements for PHLAME Example

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

	Estimates	S.E.	Est./S.E.
Effects from TEAM to BMI2			
Sum of indirect	-0.013	0.010	-1.283
Specific indirect			
BMI2			
TOTAL2			
DIETSUP2			
TEAM	-0.013	0.010	-1.283
Effects from TEAM to TOTAL2			
Sum of indirect	0.197	0.084	2.347
Specific indirect			
TOTAL2			
DIETSUP2			
TEAM	0.197	0.084	2.347

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Mediation in Structural Equation Models

- Many models have multiple IVs, DVs, and/or mediators
- With more than one dependent variable, a more detailed modeling approach is required. Separate regression analyses will not longer be accurate, e.g., correlated dependent variables, measurement models, correlated error terms. The new method is called path analysis, structural equation modeling, or covariance structure modeling.
- Matrices are used to specify and estimate these models because matrices provide a way to organize all the relations in the model. The number and type of mediated effects are increased in these models. Matrix equations are used to find mediated effects and their standard errors.

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Three Ways to Describe a Model

- Verbal Description: Translation of ideas often the hardest part of SEM.
- Diagram: Amos, for example, takes a diagram as input to the program, i.e., the model is specified with a diagram.
- Equations: Can be used for all programs. Equations may be regular regression equations or the matrix representation of equations. With complex models, the matrices provide a general way to represent effects.

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Model Specification

- Model specification includes:
- Specifying variables to be included in the model
- Specifying the number of latent variables, if any, to be modeled (no latent variables in Path Analysis)
- Specifying the relations between the variables
- Specifying constraints, etc.

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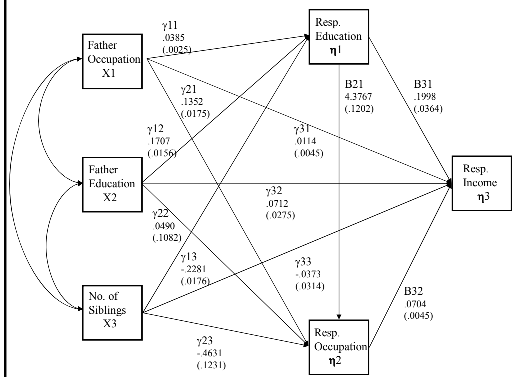
Socioeconomic Status and Achievement

- Duncan et al. (1972) presented data on achievement that have been used to illustrate methodological developments in mediation. The data are from 3214, 35-44 year old males measured during the March of 1962 as part of a large survey of the civilian labor force.
- Occupational Changes in a Generation (OCG) Study.
- There are six variables: X1 father's occupation, X2 father's education, X3 number of siblings in the respondent's family, Y1 respondent's education, Y2 respondent's occupational status, and Y3 respondent's income.
- Many types of mediated effects focusing on parental effects on offspring.

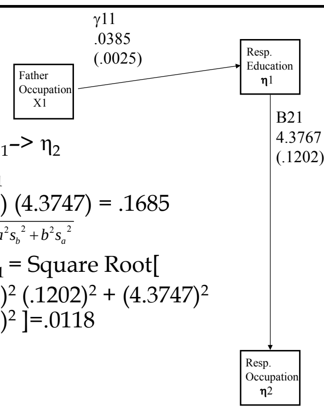


Equations in the OCG Model

$$\eta_1 = \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \gamma_{13}\xi_3 + \zeta_1 \qquad (6.19)$$

$$\eta_2 = \beta_{21}\eta_1 + \gamma_{21}\xi_1 + \gamma_{22}\xi_2 + \gamma_{23}\xi_3 + \zeta_2 \qquad (6.20)$$

$$\eta_3 = \beta_{31}\eta_1 + \beta_{32}\eta_2 + \gamma_{31}\xi_1 + \gamma_{32}\xi_2 + \gamma_{33}\xi_3 + \zeta_3 \qquad (6.21)$$



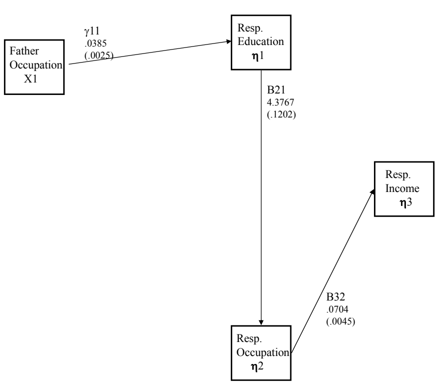
X_1->η_1-> η_2

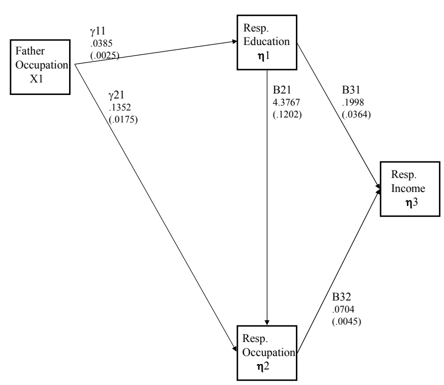
$\gamma_{11}\beta_{21}$

(.0385) (4.3747) = .1685

$$s_{ab} = \sqrt{a^2 s_b^2 + b^2 s_a^2}$$

$s_{\gamma 11\beta 21}$ = Square Root[$(.0385)^2 (.1202)^2 + (4.3747)^2 (.0025)^2$]=.0118





Mplus OCG Program

```
TITLE: Chapter 6 Multiple Mediator Model Path Analysis with OCG Data
DATA:
 Type is STD Correlation; *Reads in standard deviations and correlation matrix.
 Ngroups=1;
 Nobservations=3214;
 File is Chap6_ocgexample.txt;
VARIABLE:*Note that mnemonic variable names are used.
 Names are INC1961 OCC1962 EDUC NUMSIB FATHOCC FATHEDUC;
 Usevariables are INC1961 OCC1962 EDUC NUMSIB FATHOCC FATHEDUC;
ANALYSIS:
 Type is general;
 Estimator is ML;
 Iterations are 1000;
 Convergence is 0.00005;
MODEL:
 INC1961 on EDUC OCC1962 FATHEDUC NUMSIB FATHOCC;
 EDUC on FATHOCC FATHEDUC NUMSIB;
 OCC1962 on FATHOCC FATHEDUC NUMSIB EDUC;
MODEL INDIRECT:*Estimates Indirect Effects.
 INC1961 ind EDUC FATHEDUC;
 INC1961 ind FATHOCC;
OUTPUT:
   sampstat mod standardized tech1 tech2;
```


Model Indirect for OCG Model

```
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS
                    Estimates   S.E.  Est./S.E.   Std    StdYX
Effects from FATHOCC to INC1961                         *MODEL INDIRECT INC1961 IND FATHOCC;

 Total               0.040    0.005    8.734    0.040   0.175
 Total indirect      0.029    0.002   13.334    0.029   0.125
 Specific indirect
  INC1961
  OCC1962
  FATHOCC            0.010    0.001    6.930    0.010   0.041
  INC1961
  EDUC
  FATHOCC            0.008    0.001    5.180    0.008   0.033
  INC1961
  OCC1962
  EDUC
  FATHOCC            0.012    0.001   10.561    0.012   0.051  *Three path mediated effect
 Direct
  INC1961
  FATHOCC            0.011    0.004    2.535    0.011   0.049

Effects from FATHEDUC to INC1961                        *MODEL INDIRECT INC1961 IND FATHEDUC;
 Sum of indirect     0.034    0.007    4.912    0.034   0.024
 Specific indirect
  INC1961
  EDUC
  FATHEDUC           0.034    0.007    4.912    0.034   0.024
```


Summary of OCG Indirect Effects

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Table 6.15 Specific Indirect Effects and Standard Errors for the Achievement Model

Effect	Parameter	Estimate	Standard Error
$\xi_1 \rightarrow \eta_1 \rightarrow \eta_2$	$\gamma_{11}\beta_{21}$	0.1685	0.0118
$\xi_1 \rightarrow \eta_2 \rightarrow \eta_3$	$\gamma_{21}\beta_{32}$	0.0095	0.0014
$\xi_1 \rightarrow \eta_1 \rightarrow \eta_3$	$\gamma_{11}\beta_{31}$	0.0077	0.0015
$\xi_1 \rightarrow \eta_1 \rightarrow \eta_2 \rightarrow \eta_3$	$\gamma_{11}\beta_{21}\beta_{32}$	0.0119	0.0011
$\xi_2 \rightarrow \eta_1 \rightarrow \eta_2$	$\gamma_{12}\beta_{21}$	0.7471	0.0713
$\xi_2 \rightarrow \eta_2 \rightarrow \eta_3$	$\gamma_{22}\beta_{32}$	0.0035	0.0076
$\xi_2 \rightarrow \eta_1 \rightarrow \eta_3$	$\gamma_{12}\beta_{31}$	0.0341	0.0070
$\xi_2 \rightarrow \eta_1 \rightarrow \eta_2 \rightarrow \eta_3$	$\gamma_{12}\beta_{21}\beta_{32}$	0.0526	0.0060
$\xi_3 \rightarrow \eta_1 \rightarrow \eta_2$	$\gamma_{13}\beta_{21}$	−0.9983	0.0818
$\xi_3 \rightarrow \eta_1 \rightarrow \eta_3$	$\gamma_{13}\beta_{31}$	−0.0456	0.0090
$\xi_3 \rightarrow \eta_2 \rightarrow \eta_3$	$\gamma_{23}\beta_{32}$	−0.0326	0.0089
$\xi_3 \rightarrow \eta_1 \rightarrow \eta_2 \rightarrow \eta_3$	$\gamma_{13}\beta_{21}\beta_{32}$	−0.0703	0.0073
$\eta_1 \rightarrow \eta_2 \rightarrow \eta_3$	$\beta_{21}\beta_{32}$	0.3081	0.0214

Anabolic Steroid Book Example

X three measures of coach tolerance for steroids at time 1. Higher means less coach tolerance.

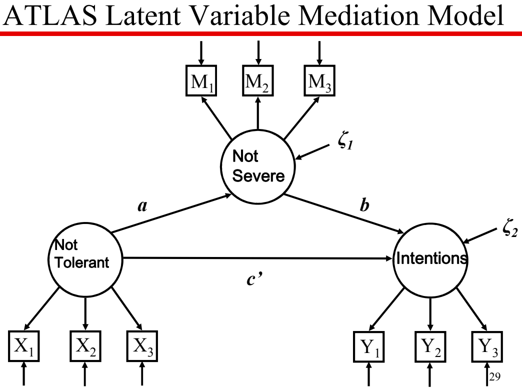
M three measures of perceived severity of steroid use at time 2. Higher means less perceived severity.

Y three measures of intentions to use steroids at time 3. Higher means more intentions to use.

Same mediation relations relating the now latent X, M, and Y variables, X to M, M to Y, and X to Y.

N=547

ATLAS Latent Variable Mediation Model



Mplus for Latent Variable Mediation Model

```
TITLE:  Chapter 7 Three Factor Latent Variable Model;
 DATA:  FILE IS chap7_exp1.txt;    TYPE IS CORRELATION STD;
    NGROUPS = 1;    NOBSERVATIONS = 547;
   VARIABLE: NAMES ARE coach1 coach2 coach3 severe1 severe2 severe3 intent1
    intent2 intent3;  USEVARIABLES ARE coach1 coach2 coach3 severe1 severe2
     severe3 intent1  intent2 intent3;
ANALYSIS:    TYPE IS GENERAL;    ESTIMATOR IS ML;
Model:   coach by coach1@1 coach2 coach3;
           severe by severe1@1 severe2 severe3;
           intent by intent1@1 intent2 intent3;
           intent on severe coach;
           severe on coach;
  Model indirect:
  intent ind coach;
  OUTPUT:  SAMPSTAT STANDARDIZED TECH1 TECH3;
```

Mplus Output for Three Latent Variable Mediation Model

MODEL RESULTS

		Estimate	S.E.	Two-Tailed Est./S.E.	P-Value
COACH BY					
COACH1		1.000	0.000	999.000	999.000
COACH2		1.746	0.251	6.962	0.000
COACH3		1.482	0.213	6.942	0.000
SEVERE BY					
SEVERE1		1.000	0.000	999.000	999.000
SEVERE2		1.175	0.077	15.309	0.000
SEVERE3		1.269	0.082	15.455	0.000
INTENT BY					
INTENT1		1.000	0.000	999.000	999.000
INTENT2		1.470	0.069	21.429	0.000
INTENT3		1.499	0.071	20.990	0.000
INTENT ON					
SEVERE	$\hat{b}$	0.266	0.048	5.540	0.000
COACH	$\hat{c}'$	0.001	0.068	0.021	0.984
SEVERE ON					
COACH	$\hat{a}$	-0.415	0.093	-4.462	0.000

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Mplus Output for Three Latent Variable Mediation Model

TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

	Estimates	S.E.	Est./S.E.	Std	StdYX
Effects from COACH to INTENT					
Total	-0.109	0.067	-1.623	-0.084	-0.084
Total indirect	-0.110	0.031	-3.566	-0.085	-0.085
Specific indirect					
INTENT					
SEVERE					
COACH	-0.110	0.031	-3.566	-0.085	-0.085
Direct					
INTENT					
COACH	0.001	0.068	0.021	0.001	0.001

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Multivariate Delta Standard Error

For correlated *a* and *b*.

$$s_{Delta} = \sqrt{\hat{a}^2 s_{\hat{b}}^2 + \hat{b}^2 s_{\hat{a}}^2 + 2\hat{a}\hat{b}s_{\hat{a}\hat{b}}}$$

$$s_{Delta} = \sqrt{\hat{a}^2 s_{\hat{b}}^2 + \hat{b}^2 s_{\hat{a}}^2 + 2\hat{a}\hat{b}r_{\hat{a}\hat{b}}s_{\hat{a}}s_{\hat{b}}}$$

$$s_{Delta} = \sqrt{-.415^2 .048^2 + .266^2 .093^2 + 2(-.415)(.266)(.000)} = .031$$

Covariance between *a* and *b* is equal to r_{ab} times the two standard errors=.052(.093)(.048)=.000:

$$r_{\hat{a}\hat{b}} s_{\hat{a}} s_{\hat{b}} = s_{\hat{a}\hat{b}}$$

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Summary

- Many potential mediated/indirect effects in path analysis models.
- Can be complicated to calculate indirect effects and standard errors of these effects. Formulas for effects and standard errors can be applied to investigate mediation in any model.
- Mplus Model INDIRECT command is very useful for the calculation of indirect effects and their standard errors.
- Note that bootstrap and other methods can be used if the raw data are available.

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