Multiple Mediator Models (Chapter 5)

- Most behaviors are affected by multiple variables so it makes sense that there are multiple mediators.
- Straightforward extension of the single mediator case but interpretation can be more difficult especially when considering all possible relations among variables.
- The product of coefficients methods is the best way to evaluate models with multiple mediators but difference and causal step methods can work, somewhat.

Multiple Mediator Examples

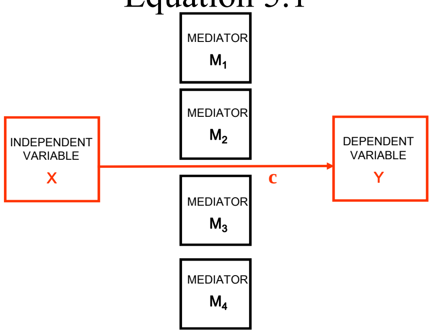
MRFIT trial targeted smoking, high cholesterol, and blood pressure to prevent heart disease.

Drug Prevention targets a host of mediators, including norms, beliefs, commitment, self-esteem, stress-management, resistance skills, communication skills.

Tobacco Cessation treatment targets tobacco withdrawal symptoms, craving, social support, beliefs about quitting.

What about surrogate endpoints? By definition is a surrogate endpoint model a single mediator model?

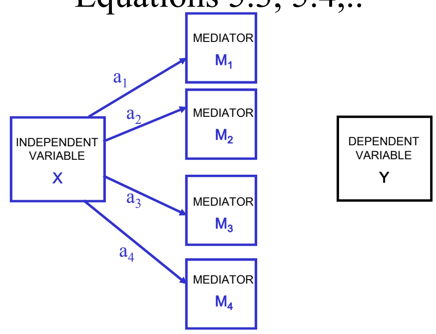
Equation 5.1



1. The independent variable is related to the dependent variable:

$\hat{Y} = \hat{c}X + \varepsilon_1$

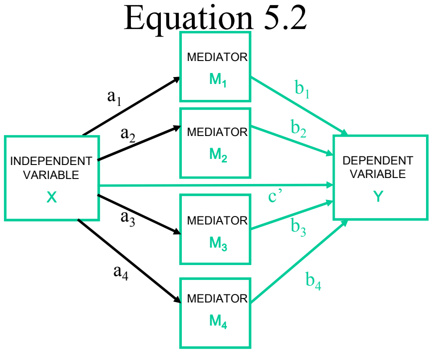
Equations 5.3, 5.4,..



2. The independent variable is related to the potential mediators:

$\hat{M}_1 = \hat{a}_1X + \varepsilon_2$, $\hat{M}_2 = \hat{a}_2X + \varepsilon_3$, $\hat{M}_3 = \hat{a}_3X + \varepsilon_4$, $\hat{M}_4 = \hat{a}_4X + \varepsilon_5$

Equation 5.2



3. The mediators are related to the dependent variable controlling for exposure to the independent variable:

$\hat{Y} = \hat{c}'X + \hat{b}_1M_1 + \hat{b}_2M_2 + \hat{b}_3M_3 + \hat{b}_4M_4 + \varepsilon_6$

Mediation Effects

Mediated effects = $a_1b_1, a_2b_2, a_3b_3, a_4b_4$

Standard error = $\sqrt{a_i^2 s_{b_i}^2 + b_i^2 s_{a_i}^2}$

Total mediated effect= $a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4 = c - c'$

Direct effect= c' Total effect= $a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4 + c' = c$

Test for significant mediation:

$$z' = \frac{a_1b_1}{\sqrt{a_i^2 s_{b_i}^2 + b_i^2 s_{a_i}^2}}$$ Compare to empirical distribution of the mediated effect

Measures of Effect Size 1

Correlation and standardized effect size measures for individual paths. Many programs output standardized effect size measures for coefficients. Correlations and partial correlations for each path are more challenging (SAS PCORR2 will produce the partial correlations squared in SAS for example).

These effect size measures for individual paths are adjusted for other variables in the model including other mediators.

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Measures of Effect Size 2

Mediated effect in terms of the change in standard deviation units of Y for a one unit change in X. Used for binary X or when one unit change is desired. (Mplus STDY)

$$OneUnitStandardized_{\hat{a}\hat{b}} = \frac{\hat{a}_i \hat{b}_i}{s_y}$$

Mediated effect in terms of the change in standard deviation units of Y for a one standard deviation change in X. Useful for continuous X. (Mplus STDXY)

$$OneSDSStandardized_{\hat{a}\hat{b}} = \frac{\hat{a}_i \hat{b}_i s_x}{s_y}$$

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Measures of Effect Size 3

Proportion Mediated = $a_i b_i/(c' + \sum a_i b_i) = a_i b_i/c$;

Ratio of Mediated to Direct Effect = $a_i b_i/c'$

Simulation studies suggest large samples are necessary for these values to be accurate for the single mediator model, e.g. 500 for the proportion and 1000 for the ratio, MacKinnon et al. (1995). Absolute values do and squaring terms do not improve the situation. These may be good options for inconsistent mediation models.

R^2 mediated and k^2 proportion of the maximum mediated effect are more complicated than the single mediator model.

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Expectancy effects on Achievement

Harris and Rosenthal (1985) meta-analysis of mediators of the relation between teacher expectancy and student performance.

Here is a hypothetical study (N=40) with two mediators. (M1) social climate and (M2) material covered or input. Y is a test of achievement and X is the randomly assigned student ability value for each student. It was hypothesized that the ability score invokes an expectancy which affects warmth and material covered which leads to greater achievement.

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SAS Program for Expectancy effects on Achievement Model

```
proc reg;
model y=x;
model y=x m1 m2/covb;
model m1=x;
model m2=x;
```

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SPSS Program for Expectancy effects on Achievement Model

```
Regression
   /variables= x y m1 m2
   /dependent=y
   /enter=x.
regression
   /variables= x y m1 m2
   /statistics=defaults bcov
   /dependent=y
   /enter=x m1 m2.
regression
   /variables= x y m1
   /dependent=m1
   /enter= x.
regression
   /variables x y m2
   /dependent=m2
   /enter= x.
```

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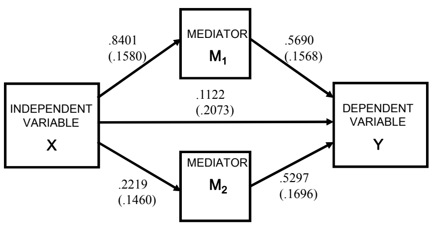
Two Mediator Model



Mediated Effect Measures

$\hat{a}_1\hat{b}_1$ = (.8401) (.5690) = .4781 for mediation through social climate and

$\hat{a}_2\hat{b}_2$ = (.2219) (.5297) = .1175 for mediation through input. The total mediated effect of $\hat{a}_1\hat{b}_1$ (.4781) plus $\hat{a}_2\hat{b}_2$ (.1175) equals .5956 which is equal to $\hat{c}-\hat{c}'$=.7078-.1122 =.5956.

The $\hat{a}_1\hat{b}_1$ mediated effect ($s_{\hat{a}_1\hat{b}_1}$ = .1499) was statistically significant ($z_{\hat{a}_1\hat{b}_1}$ = 3.183) and the $\hat{a}_2\hat{b}_2$ mediated effect ($s_{\hat{a}_2\hat{b}_2}$ = .0838) was not ($z_{\hat{a}_2\hat{b}_2}$ = 1.403).

The standard error of the total mediated effect is equal to .1717 yielding a z statistic of 3.468.

*Note that the covariance between the two mediators is not shown in the two mediator model figure to make the figure easier to read.

Confidence Limits

Mediation through social climate,

Asymmetric LCL= .195 and UCL = .825. Using the multivariate delta method standard error, LCL= .1654 and UCL = .7906.

Mediation through input,

Asymmetric LCL = -.032 and UCL = .319. Using the multivariate delta method standard error, LCL= -.0511 and UCL = . 2862.

Effect size

Social Climate Mediator
Proportion Mediated=.478/.708=.675
Standard deviation change in Y for a one unit change in X=.478/11.662=.041
Standard deviation change in Y for a one standard deviation change in X=.478*9.095/11.662=.373

Input Mediator
Proportion Mediated=.118/.708=.166
Standard deviation change in Y for a one unit change in X=.118/11.662=.010.
Standard deviation change in Y for a one standard deviation change in X=.118*9.095/11.662=.092

Test of Equality of two Mediated Effects

$$s_{\hat{a}_1\hat{b}_1-\hat{a}_2\hat{b}_2} = \sqrt{s_{\hat{a}1\hat{b}1}{}^2 + s_{\hat{a}2\hat{b}2}{}^2 - 2\hat{a}_1\hat{a}_2 s_{\hat{b}_1\hat{b}_2} - 2\hat{b}_1\hat{b}_2 s_{\hat{a}_1\hat{a}_2}}$$

$2\hat{b}_1\hat{b}_2 s_{\hat{a}_1\hat{a}_2}$ is 0 in OLS estimation of the mediation equations but this quantity should be included if there is a covariance between the two a coefficients, which may occur if covariance structure modeling is used, for example. There may also be other covariances that are needed but these are typically very small.

The difference between the two mediated effects in the expectancy example is equal to .3605 with a standard error of .1717 yielding a z statistic of 2.099.

Contrasts can be used to test pairs of mediated effects in any model as discussed later (see MacKinnon (2000) Contrasts in Multiple Mediator Models).

Other Tests for Mediation in the Multiple Mediator Model

Product of coefficients will generalize to other models.

Causal Steps

Joint Significance

Difference in Coefficients

Baron & Kenny and Judd & Kenny, test for the Multiple Mediator Model

1. X is significantly related to Y.
2. X must affect M1 and X must affect M2.
3. M1 and M2 must affect Y after adjustment for X.
4. $\hat{c}'$ Must be nonsignificant for JK or $\hat{c}'$ must be less than $\hat{c}$ for BK.

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Limitations of BK and JK Steps: Multiple Mediator Model

1. Just investigates overall mediation; no way to look at specific mediated effects.
2. No significance testing of specific or total mediated effects. What if path from M1 to Y is statistically significant but M2 to Y is not? What if X to M1 is significant but X to M2 is not?
3. Requirement of a significant total effect is not necessary for the same reasons as for the single mediator case, e.g., inconsistent mediation effects.

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Limitations of Joint Significance Causal Steps test

1. Could look at significance of two paths in each mediation relation.
2. But the total mediated effect is interesting too, not just each path in the specific mediated effect. Cumbersome to test significance of the total effect with a joint significance test, perhaps a null hypothesis of whether all four paths (in the two mediator model) are statistically significant.

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Limitations of Difference in Coefficients Mediation Test

1. Provides a test of the overall mediated effect $\hat{c}-\hat{c}'$ with and its standard error.
2. No clear way to get estimates of the specific mediated effects. Could test prediction of Y with just M1, then Y with just M2, and Y with both M1 and M2, and use the change in coefficients in some way to get an estimate of each specific mediated effect.
3. The method ignores the individual *a* paths which are important to investigate specific mediated effects.

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Product of Coefficients are #1

Product of coefficients tests generate more useful information are relatively easy to apply and provide estimates, standard errors, and confidence intervals.

Product of coefficients tests apply to complex models and are used throughout the course.

Hooray for the Product of Coefficients.

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Inconsistent Mediation: Multiple Mediator Style

Inconsistent mediation models are models where at least one of the mediated effects and direct effects have different signs (see MacKinnon, Krull, & Lockwood 2000).

Same idea as in the single mediator case but it is easier to think of inconsistent mediation in the multiple mediator model because of opposing or iatrogenic effects. Iatrogenic means induced inadvertently by a physician or surgeon or by medical treatment or diagnostic procedures.

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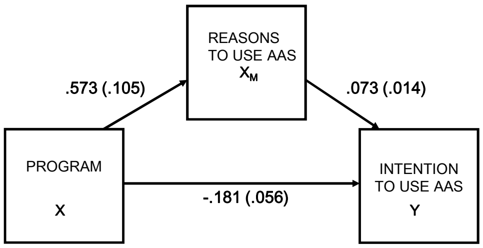
Inconsistent mediation in ATLAS Data



Mediated effect = .042
Standard error = .011

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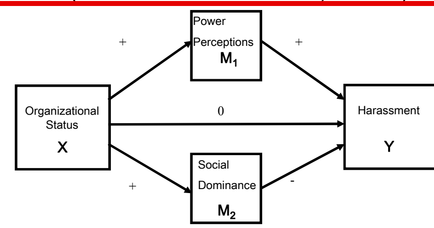
Mediators of null effect of status on perceived sexual harassment (Sheets & Braver,1999)



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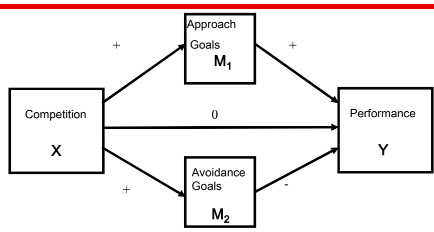
Mediators of the competition effect (Murayama & Elliot, 2012)



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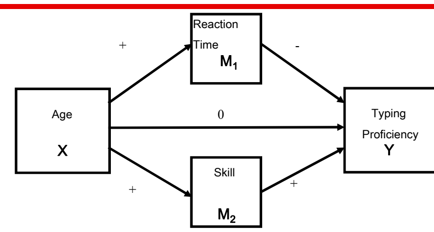
Mediators of the null effect of age on typing (Salthouse, 1984)



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Selection, Optimization, and Compensation Theory of Aging (Baltes, 1997)

A theory predicting inconsistent mediation.

Selection - restriction of life to fewer domains of functioning.

Optimization - selection of behaviors that enrich or augment basic reserves of focus on life course.

Compensation - compensate for loss of capacity with other methods. Compensation implies opposing mediational processes for the effect of aging.

Baltes, P. B. (1997). On the incomplete architecture of human ontogeny: Selection, optimization, and compensation as foundation of developmental theory. *American Psychologist, 52,* 366–380.

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Contrasts in Multiple Mediator Models

Multiple mediator models introduce more than one mediated effect for each dependent variable.

Contrasts may used to compare pairs of effects or two groups of mediated effects.

The direct effect may be included in contrasts also.

Any combination of effects may be compared as long as all effects have the same dependent variable – makes scaling of all effects the same and thus they may be directly compared to one another.

(MacKinnon, 2000)

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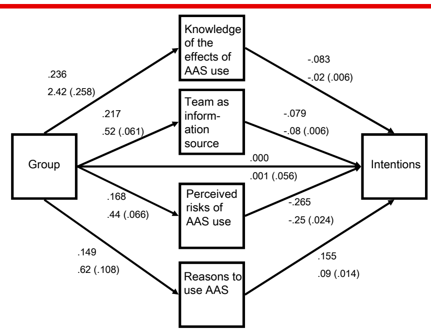
Multiple Mediator Model of Intent to Use Anabolic Steroids



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Summary of Multiple Mediators

Remember the assumptions of the single mediator model apply to the multiple mediator model. The additional variables address the omitted variable assumption. But other assumptions still apply.

Specificity of significant mediation paths improve interpretation.

The results from a multiple mediator model may shed light on the true underlying mechanisms but there are alternative explanations of results. Remember that the path relating the mediators to Y is correlational.

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