Multilevel Models for Complex Clustering

Cross-Classification and Multiple Memberships

Clustered Data Structures

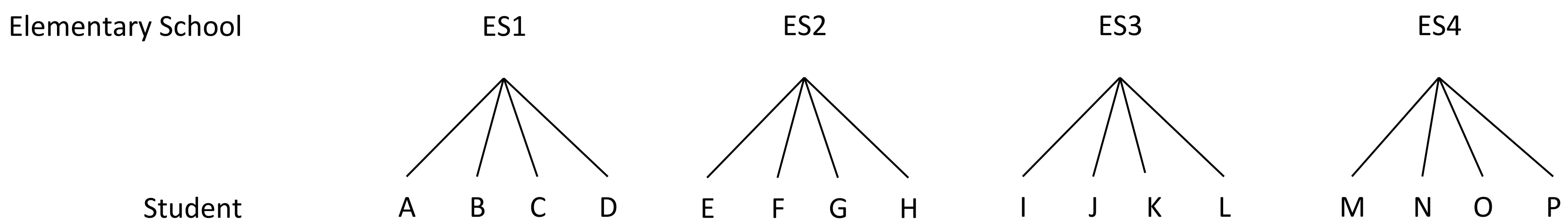
Clustered Data Structures

- Most data are hierarchical in nature.
  - *Lower level units are clustered in higher level units*

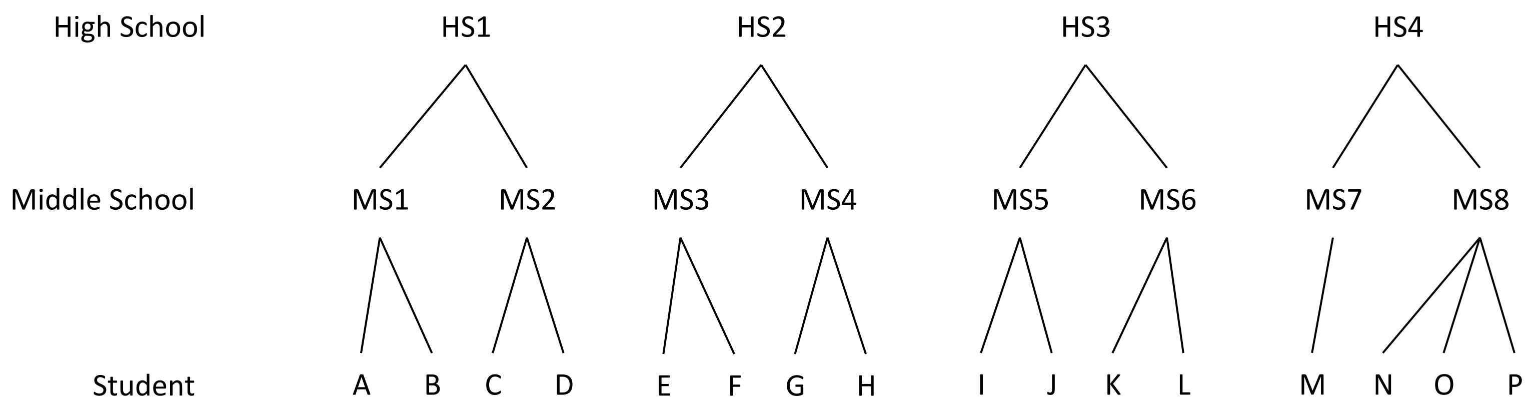
- Examples:
  - Patients clustered in hospitals
  - *Students clustered in schools*
  - Students clustered in classrooms
  - Repeated measures clustered in persons

Clustered Data Structures

- Examples of clustering in education data:
  - *Students in Elementary Schools:*



  - *Students in Middle Schools then High Schools:*



Clustered Data Structures

- Multilevel models are good for clustered data.

- But, traditional multilevel models assume *pure clustering* of lower level units in higher level units.
  - *e.g. Pure clustering of students in schools.*

Clustered Data Structures

- Today I'm going to talk about models that don't require *pure clustering* of lower level units in higher level units.

- Specifically, I'll discuss:
  - *Multiple membership random effects models, (MMREMs).*
  - *Cross-classified random effects models, (CCREMs).*

Multiple Membership Random Effects Models

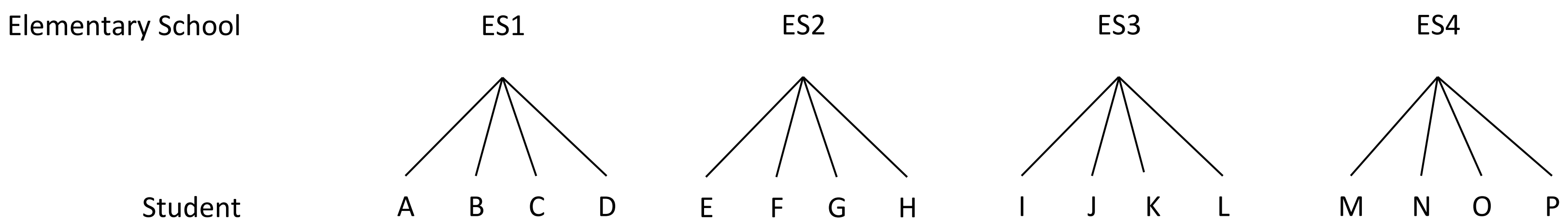
Pure Clustering

vs.

Multiple Membership

Clustered Data Structures

- Students clustered in schools:
  - *Network graph*



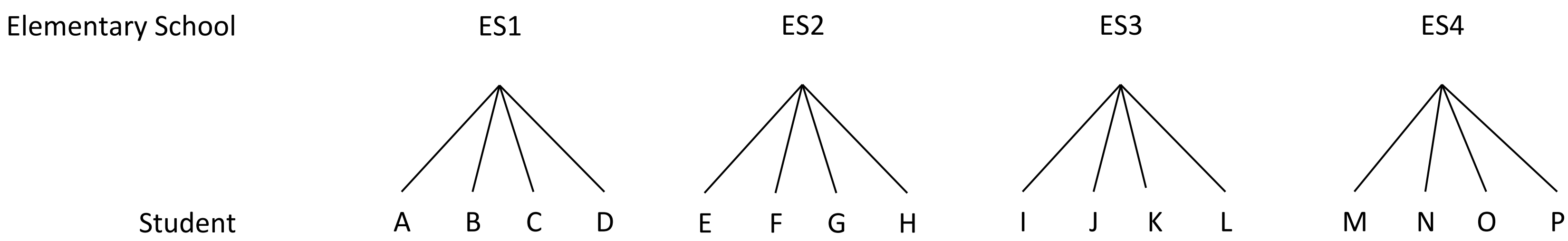
  - *Network table*

School			
1	**2**	**3**	**4**
A,B,C,D			
	E,F,G,H		
		I,J,K,L	
			M,N,O,P

Pure Clustering

- Pure clustering of students in schools
  - *What makes the clustering pure?*

  - Each student attends a single school.



  - Each lower-level unit is a member of a single higher-level unit.

Pure Clustering

– Suppose I know the school(s) each student attended in the fall and spring semesters.

	Spring School			
	1	**2**	**3**	**4**
Fall School				
1	A,B,C,D			
2		E,F,G,H		
3			I,J,K,L	
4				M,N,O,P

– Each student's fall school is the same as their spring school.

- *Students are purely clustered in schools.*

Pure Clustering

- If there's *pure clustering*, I can fit the following traditional multilevel model:

$$Y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$$

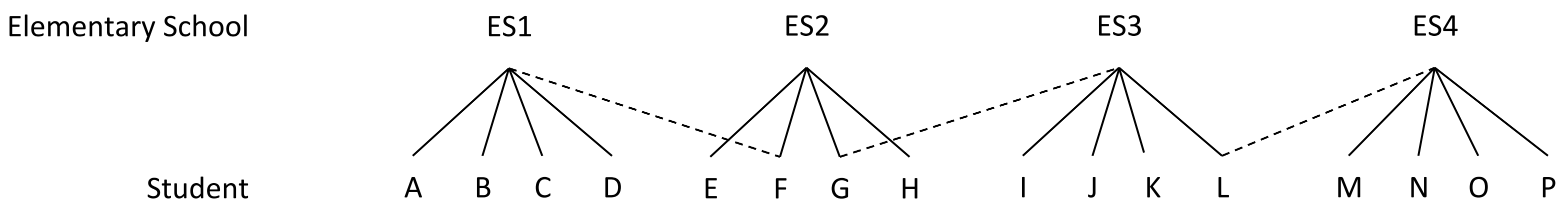
- Where *i* indexes the student and *j* indexes the school and: $u_{0j} \sim N(0, \tau_{00})$ and $e_{ij} \sim N(0, \sigma^2)$

- The intra-class correlation is given as:

$$ICC = \frac{\tau_{00}}{\tau_{00} + \sigma^2}$$

Impure Clustering

- *What would make the clustering impure?*
  - If some students attended multiple schools:



  - Students *F*, *G* and *L* attend multiple schools in this case.
  - This is an example of *multiple membership*

Impure Clustering

– In this case, the fall and spring schools differ for some children.

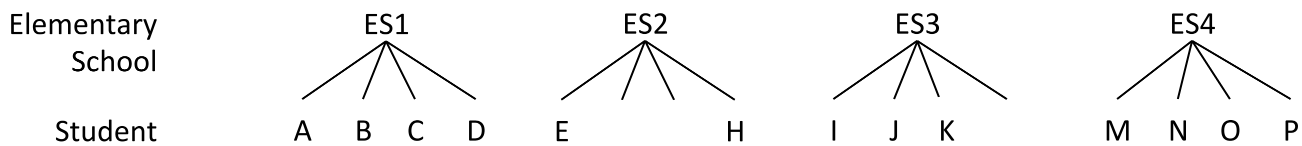
	Spring School			
	1	**2**	**3**	**4**
Fall School				
1	A,B,C,D			
2	F	E,H	G	
3			I,J,K	L
4				M,N,O,P

– Students *F*, *G* and *L* are *mobile students.*

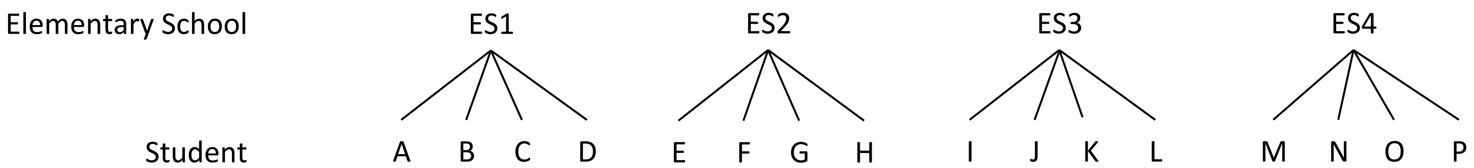
- *i.e. They are members of multiple schools.*

Impure Clustering

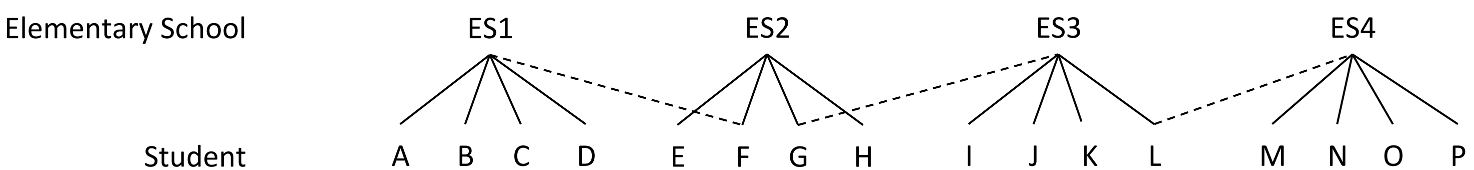
- Options for dealing with mobile students:
  - *Delete* mobile students from the dataset



  - *Ignore* mobility



  - Fit a *MMREM*

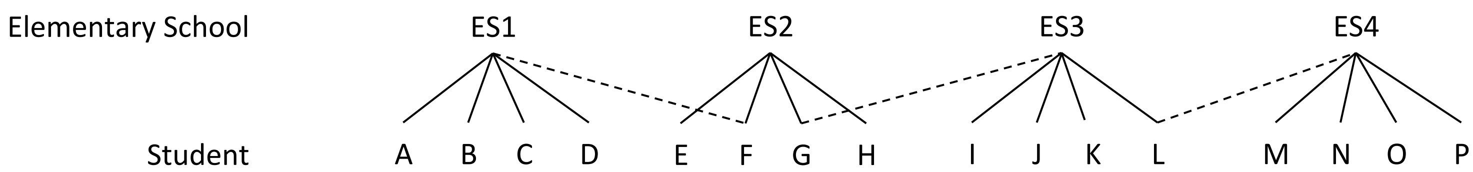


MMREM Example 1

Estimating Gender and Charter School Effects

MMREM

- Suppose students could attend, at most, 2 schools during a data collection period.



- A two-level MMREM can be given as follows:

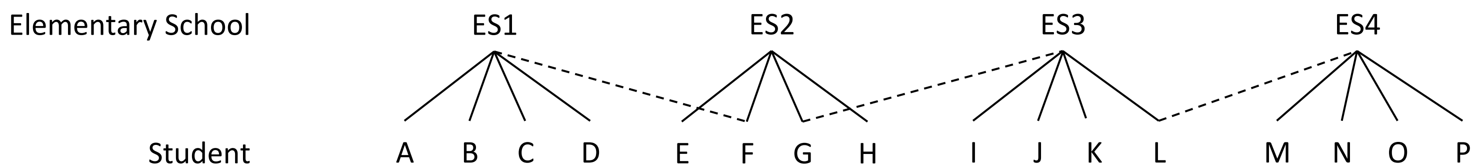
$$Y_{i\{j\}} = \gamma_{00} + w_{ij_1} u_{0j_1} + w_{ij_2} u_{0j_2} + e_{i\{j\}}$$

Weight assigned to the random effect of school j_1

Weight assigned to the random effect of school j_2

MMREM

- Consider student *F* who attended *ES1* and *ES2*



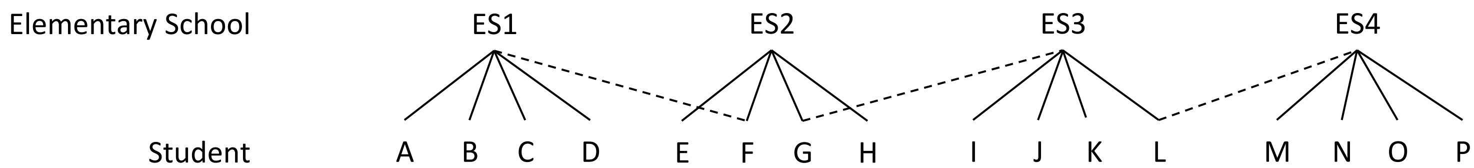
- His outcome could be modeled as follows:

$$Y_{F\{ES1,ES2\}} = \gamma_{00} + 0.50 * u_{0ES1} + 0.50 * u_{0ES2} + e_{F\{ES1,ES2\}}$$

- I've given equal weight to the effects of *ES1* and *ES2*.

MMREM

- Suppose we knew student *F* attended *ES1* for twice as long as *ES2*



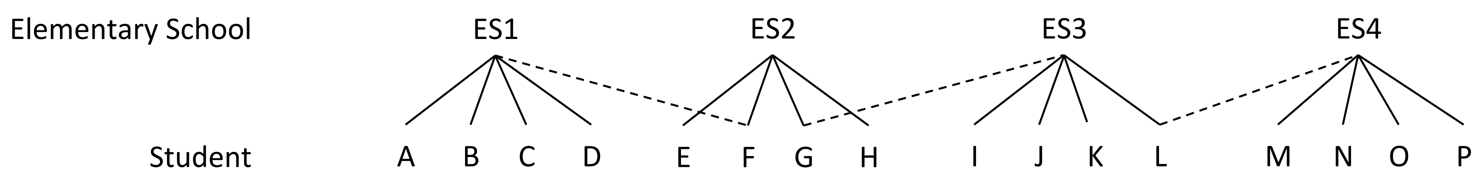
- His outcome could be modeled as follows:

$$Y_{F\{ES1,ES2\}} = \gamma_{00} + 2/3 * u_{0ES1} + 1/3 * u_{0ES2} + e_{F\{ES1,ES2\}}$$

- I've chosen weights that reflect the amount of time the student spent in each school.

MMREM

- Consider student *A* who attended *ES1* only:



- His outcome could be modeled as follows:

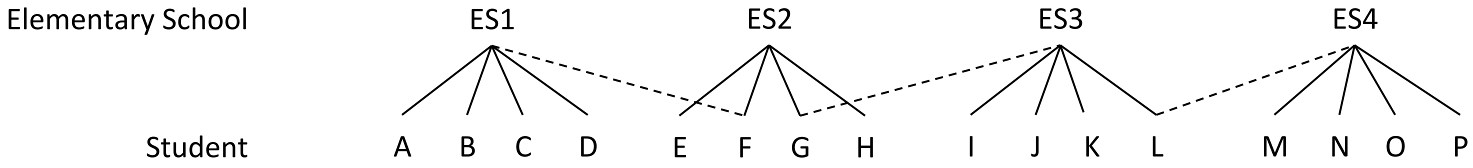
$$Y_{A\{ES1\}} = \gamma_{00} + 1.0 * u_{0ES1} + 0.0 * u_{0NA} + e_{A\{ES1\}}$$

- The MMREM simplifies to a traditional MLM when the student attends a single school.

MMREM

Example data:

Student	Y	School 1	School 2	Weight 1	Weight 2
A	100	1	1	1	0
…					
F	95	1	2	0.5	0.5
G	90	2	3	0.5	0.5
…					



MMREM

- Suppose students could attend, at most, 4 schools during a data collection period.
  - *i.e. Some students attend 4 different schools.*

- The MMREM could be given as follows:

$$Y_{i\{j\}} = \gamma_{00} + w_{ij_1}u_{0j_1} + w_{ij_2}u_{0j_2} + w_{ij_3}u_{0j_3} + w_{ij_4}u_{0j_4} + e_{i\{j\}}$$

- More weights are added but the weights should still sum to 1.

MMREM

- For someone who attended 4 different schools:

$$Y_{i\{j\}} = \gamma_{00} + 1/4u_{0j_1} + 1/4u_{0j_2} + 1/4u_{0j_3} + 1/4u_{0j_4} + e_{i\{j\}}$$

- And for someone who attended only 1 school:

$$Y_{i\{j\}} = \gamma_{00} + 1*u_{0j_1} + 0*u_{0NA} + 0*u_{0NA} + 0*u_{0NA} + e_{i\{j\}}$$

- Which simplifies to a traditional MLM:

$$Y_{i\{j\}} = \gamma_{00} + u_{0j_1} + e_{i\{j\}}$$

MMREM

- We can condense the following notation:

$$Y_{i\{j\}} = \gamma_{00} + w_{ij_1} u_{0j_1} + w_{ij_2} u_{0j_2} + w_{ij_3} u_{0j_3} + w_{ij_4} u_{0j_4} + e_{i\{j\}}$$

- To:

$$Y_{i\{j\}} = \gamma_{00} + \sum_{h \in \{j\}} w_{ih} u_{0h} + e_{i\{j\}}$$

- Where the weights always sum to 1:

$$\sum_{h \in \{j\}} w_{ih} = 1$$

MMREM

- So, an unconditional MMREM is typically given as:

$$Y_{i\{j\}} = \gamma_{00} + \sum_{h \in \{j\}} w_{ih} u_{0h} + e_{i\{j\}}$$

- Where: $u_{0h} \sim N(0, \tau_{00})$ *and* $e_{i\{j\}} \sim N(0, \sigma^2)$

- And the intra-class correlation is given as:

$$ICC = \frac{\tau_{00}}{\tau_{00} + \sigma^2}$$

MMREM

Real Data Example

- I have a reading test score for 3,022 students.

- The students attended as many as 4 schools across the data collection period.

1 School: n = 2,496

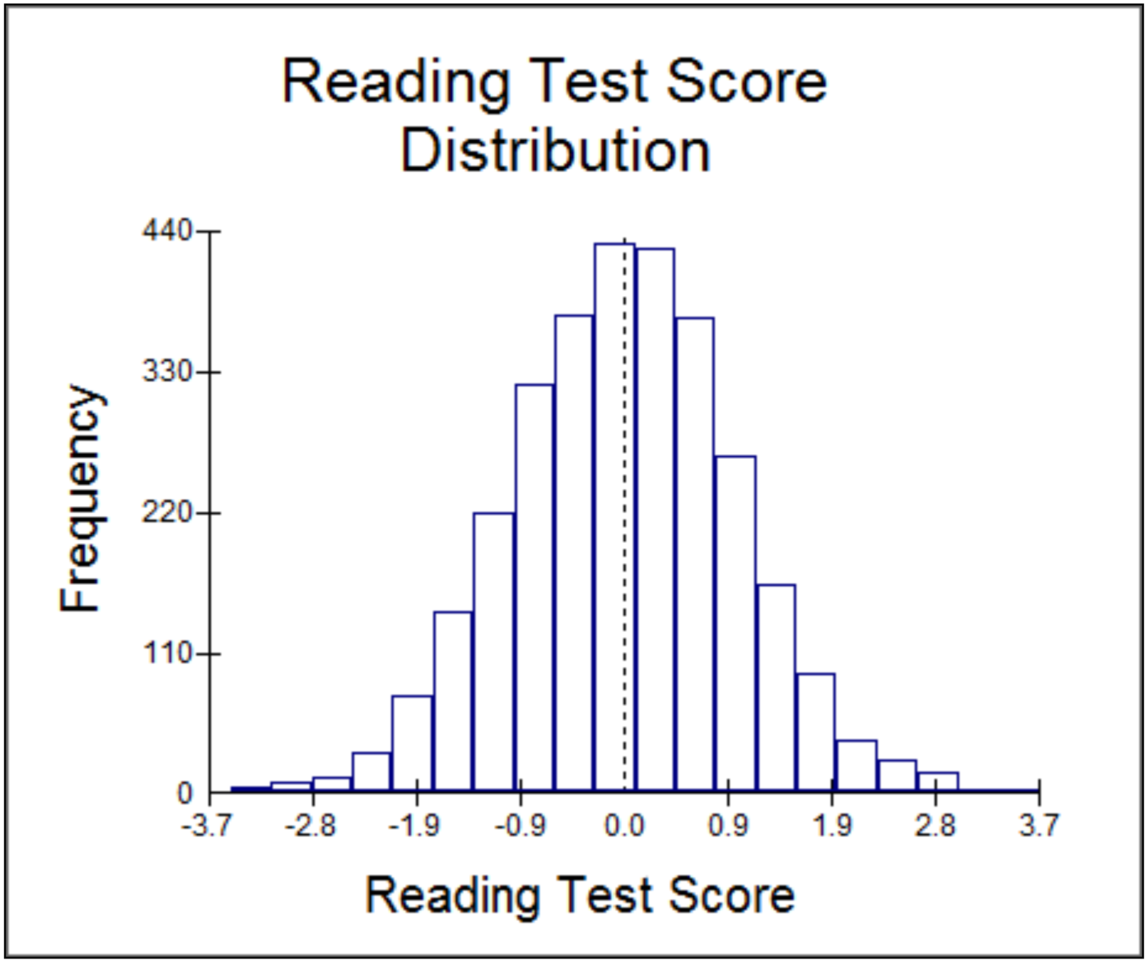
2 Schools: n = 472

3 Schools: n = 52

4 Schools: n = 2

MMREM

- Here are the reading test scores:



MMREM

- I'm going to fit the following unconditional MMREM:

$$Y_{i\{j\}} = \gamma_{00} + w_{ij_1}u_{0j_1} + w_{ij_2}u_{0j_2} + w_{ij_3}u_{0j_3} + w_{ij_4}u_{0j_4} + e_{i\{j\}}$$

- I want to determine the proportion of the variance in reading test scores that is *between schools*.

$$ICC = \frac{\tau_{00}}{\tau_{00} + \sigma^2}$$

MMREM

- Results, (*from MLwiN*):

$$\text{Reading}_i \sim \mathrm{N}(XB,\ \Omega)$$

$$\text{Reading}_i = \beta_{0i}\text{cons}_i$$

$$\beta_{0i} = -0.015(0.040) + \sum_{j \in school1(i)} w_{i,j}^{(2)} u_{0j}^{(2)} + e_{0i}$$

τ_{00}

$$\left[u_{0,school1(i)}^{(2)}\right] \sim \mathrm{N}(0,\ \Omega_u^{(2)})\ :\ \Omega_u^{(2)} = \left[0.168(0.026)\right]$$

$$\left[e_{0i}\right] \sim \mathrm{N}(0,\ \Omega_e)\ :\ \Omega_e = \left[0.838(0.022)\right] \longleftarrow \sigma^2$$

$$\textit{Deviance(MCMC)} = 8041.698(3022 \text{ of } 3022 \text{ cases in use})$$

- ICC = .168/(.168 + .838) = 0.17

MMREM

- Suppose I *ignore* student mobility and acknowledge only the *FIRST* school that a student attended.

- This model would be given as follows:

$$Y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$$

- This is just a traditional MLM using only the initially attended school.

MMREM

- *Example data:*

Student	Y	School 1
A	100	1
…		
F	95	1
G	90	2
…		

- Here we only acknowledge the initially attended school.

MMREM

- Results:

$$\text{Reading}_i \sim \text{N}(XB,\ \Omega)$$

$$\text{Reading}_i = \beta_{0i}\text{cons}_i$$

$$\beta_{0i} = -0.006(0.039) + u^{(2)}_{0,school1(i)} + e_{0i}$$

$$\left[u^{(2)}_{0,school1(i)}\right] \sim \text{N}(0,\ \Omega^{(2)}_u)\ :\ \Omega^{(2)}_u = \left[0.151(0.024)\right]$$

$$\left[e_{0i}\right] \sim \text{N}(0,\ \Omega_e)\ :\ \Omega_e = \left[0.849(0.022)\right]$$

Deviance(MCMC) = 8082.605(3022 of 3022 cases in use)

- ICC = .151/(.151 + .849) = 0.15

MMREM

- Failure to model multiple membership typically results in an *underestimate of the higher-level variance term*:

	School Variance τ_{00}	Student Variance σ^2	*DIC*
MMREM	0.168	0.838	8041
MLM	0.151	0.849	8082

- And a worse fitting model.
  - *i.e. Higher DIC value.*

MMREM

- I could add the student-level predictor *Gender* to the MMREM:

$$Y_{i\{j\}} = \gamma_{00} + \gamma_{10} Gender_{i\{j\}} + \sum_{h \in \{j\}} w_{ih} u_{0h} + e_{i\{j\}}$$

- Where the *Gender* effect is modeled as fixed across schools:

MMREM

- We could also add the school-level variable charter school status, (*Charter*).

$$Y_{i\{j\}} = \gamma_{00} + \gamma_{10} Gender_{i\{j\}} + \gamma_{01} \sum_{h \in \{j\}} w_{ih} Charter_h + \sum_{h \in \{j\}} w_{ih} u_{0h} + e_{i\{j\}}$$

- We acknowledge that some mobile students attended both charter and non-charter schools.

MMREM

- Suppose a student attended *two schools* for equal amounts of time.

- We could express the following MMREM:

$$Y_{i\{j\}} = \gamma_{00} + \gamma_{10} Gender_{i\{j\}} + \gamma_{01} \sum_{h \in \{j\}} w_{ih} Charter_h + \sum_{h \in \{j\}} w_{ih} u_{0h} + e_{i\{j\}}$$

- As:

$$Y_{i\{j\}} = \gamma_{00} + \gamma_{10} Gender_{i\{j\}} + \gamma_{01} (0.5 * \underset{\uparrow \text{ } j_1 \text{ charter status}}{Charter_{j_1}} + 0.5 * \underset{\uparrow \text{ } j_2 \text{ charter status}}{Charter_{j_2}})$$
$$+ 0.5 * u_{0j_1} + 0.5 * u_{0j_2} + e_{i\{j\}}$$

MMREM

- *Example data:*

Student	Y	School 1	School 2	Weight 1	Weight 2	Charter 1	Charter 2	Gender
A	100	1	1	1	0	1	1	1
…								
F	95	1	2	0.5	0.5	1	0	0
G	90	2	3	0.5	0.5	0	0	0
…								

- The model for student *F* would be given as:

$$Y_{i\{j\}} = \gamma_{00} + \gamma_{10} Gender_{i\{j\}} + \gamma_{01}(0.5 * \underbrace{1.0}_{\text{Charter 1}} + 0.5 * \underbrace{0.0}_{\text{Charter 2}})$$
$$+ 0.5 * u_{0_{j_1}} + 0.5 * u_{0_{j_2}} + e_{i\{j\}}$$

MMREM

- *Example data:*

Student	Y	School 1	School 2	Weight 1	Weight 2	Charter 1	Charter 2	Gender
A	100	1	1	1	0	1	1	1
…								
F	95	1	2	0.5	0.5	1	0	0
G	90	2	3	0.5	0.5	0	0	0
…								

- The model for student *A* would be given as:

$$Y_{i\{j\}} = \gamma_{00} + \gamma_{10} Gender_{i\{j\}} + \gamma_{01}(\underset{\text{Weight 1}}{1.0} * 1 + \underset{\text{Weight 2}}{0.0} * 1)$$

$$+ 1.0 * u_{0j_1} + 0.0 * u_{0NA} + e_{i\{j\}}$$

MMREM

- Results:

Gender effect

$$\text{Reading}_i \sim \text{N}(XB,\ \Omega)$$

$$\text{Reading}_i = \beta_{0i}\text{cons}_i + -0.437(0.033)\text{Gender}_i + -0.077(0.093)\text{Charter}_i$$

Charter effect

$$\beta_{0i} = 0.184(0.050) + \textstyle\sum_{j \in school1(i)} w_{i,j}^{(2)} u_{0j}^{(2)} + e_{0i}$$

$$\begin{bmatrix} u_{0,school1(i)}^{(2)} \end{bmatrix} \sim \text{N}(0,\ \Omega_u^{(2)}) \ : \ \Omega_u^{(2)} = \begin{bmatrix} 0.166(0.026) \end{bmatrix}$$

$$\begin{bmatrix} e_{0i} \end{bmatrix} \sim \text{N}(0,\ \Omega_e) \ : \ \Omega_e = \begin{bmatrix} 0.792(0.021) \end{bmatrix}$$

$$Deviance(MCMC) = 7870.406(3022 \text{ of } 3022 \text{ cases in use})$$

MMREM Example 2

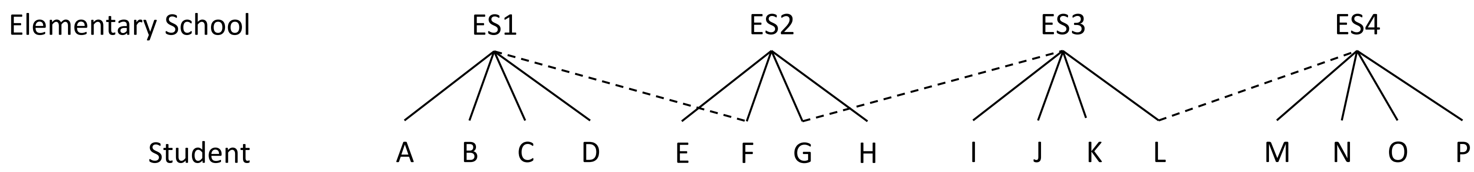
Estimating School Effects

MMREM

- Multilevel models are sometimes used to judge school performance.
  - *i.e. To estimate "school effects"*
- School effect estimates are based on how the students in a school perform on some outcome.
  - *e.g. A standardized test*
- So, it's important to have a clear idea of which students attended which schools.
  - *Student mobility is commonly ignored in these applications.*

MMREM

- Consider our illustrative data set:

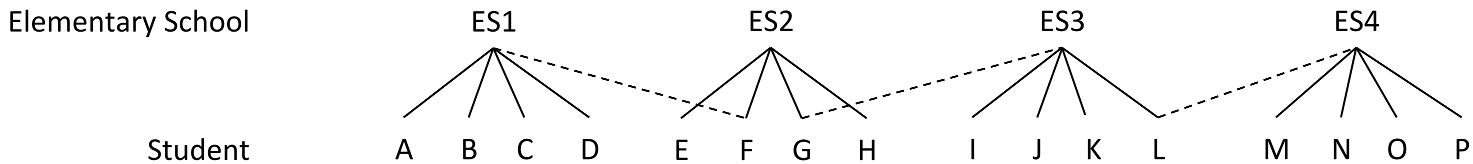


- Suppose student *F* is a really bad student.

- Should *ES1* or *ES2* get the blame for student *F*'s poor performance?
  - *A traditional MLM would assign the “blame” to only one of the schools.*

Modeling Options

- In theory, MMREMs give fairer estimates of school effects when there are mobile students.



- MMREMs acknowledge student mobility when estimating school effects.

MMREM

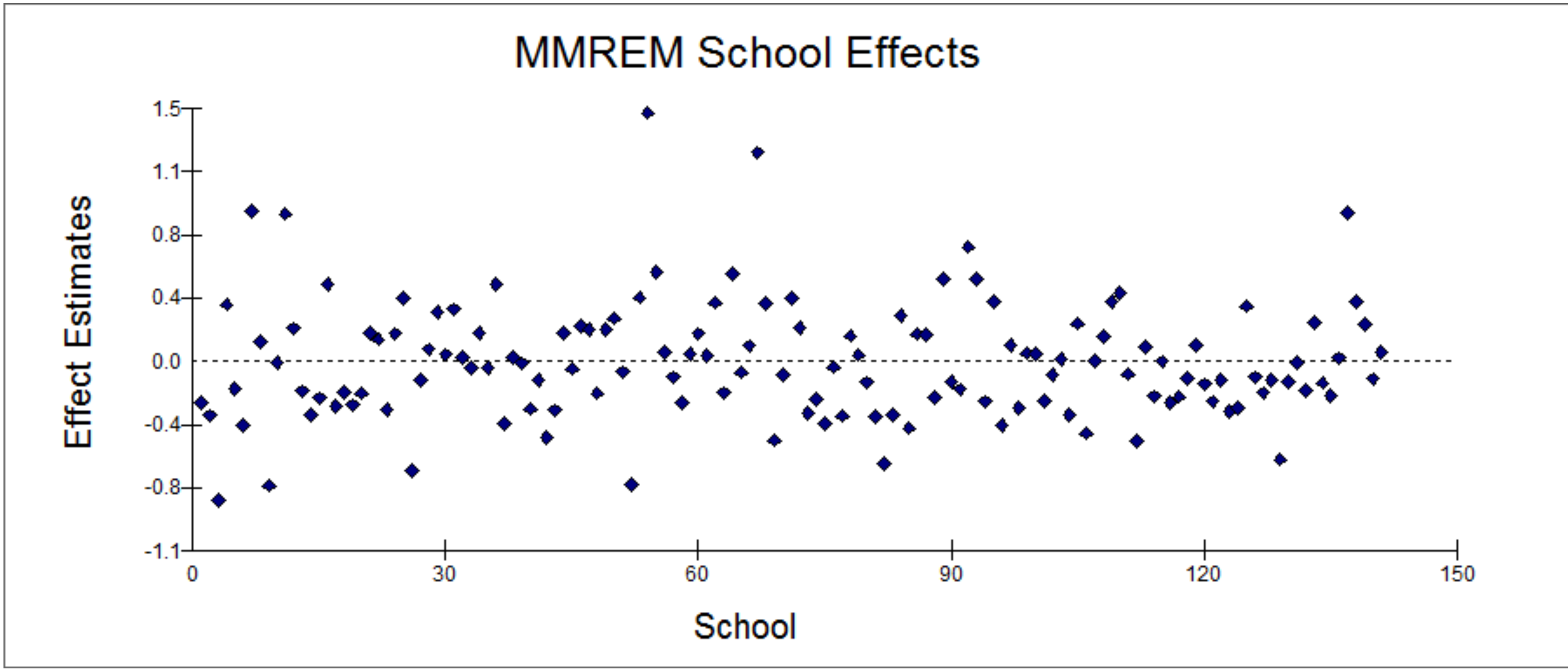
- Let's use the following MMREM to estimate school effects for the 141 schools in our dataset.

$$Y_{i\{j\}} = \gamma_{00} + \sum_{h \in \{j\}} w_{ih} \underbrace{u_{0h}}_{\text{School Effect}} + e_{i\{j\}}$$

- We'll have a school effect estimate for each school.

MMREM

- MMREM school effect estimates:



MMREM

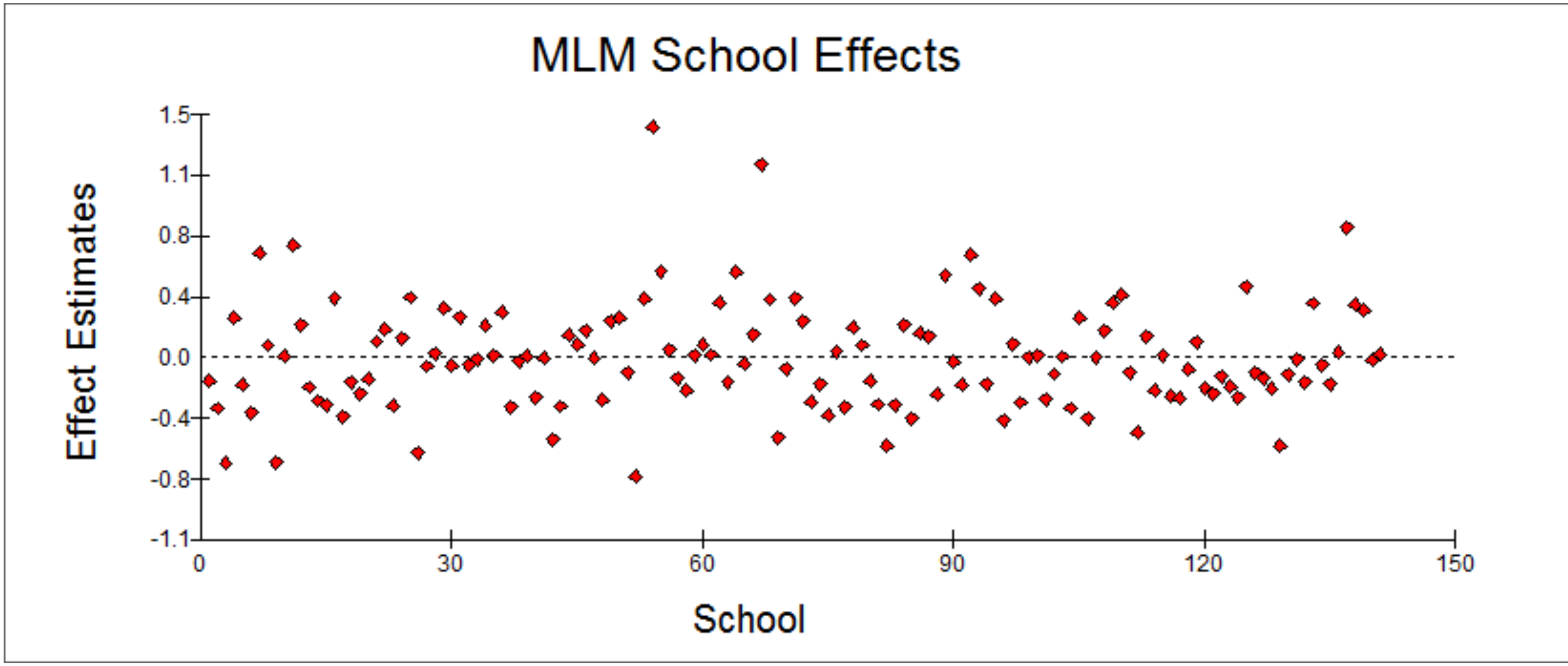
- Now let's *ignore* multiple membership and use the following MLM to estimate school effects for the 141 schools in our dataset.

$$Y_{ij_1} = \gamma_{00} + \underbrace{u_{0j_1}}_{\text{School Effect}} + e_{ij_1}$$

- These school effect estimates don't acknowledge student mobility.

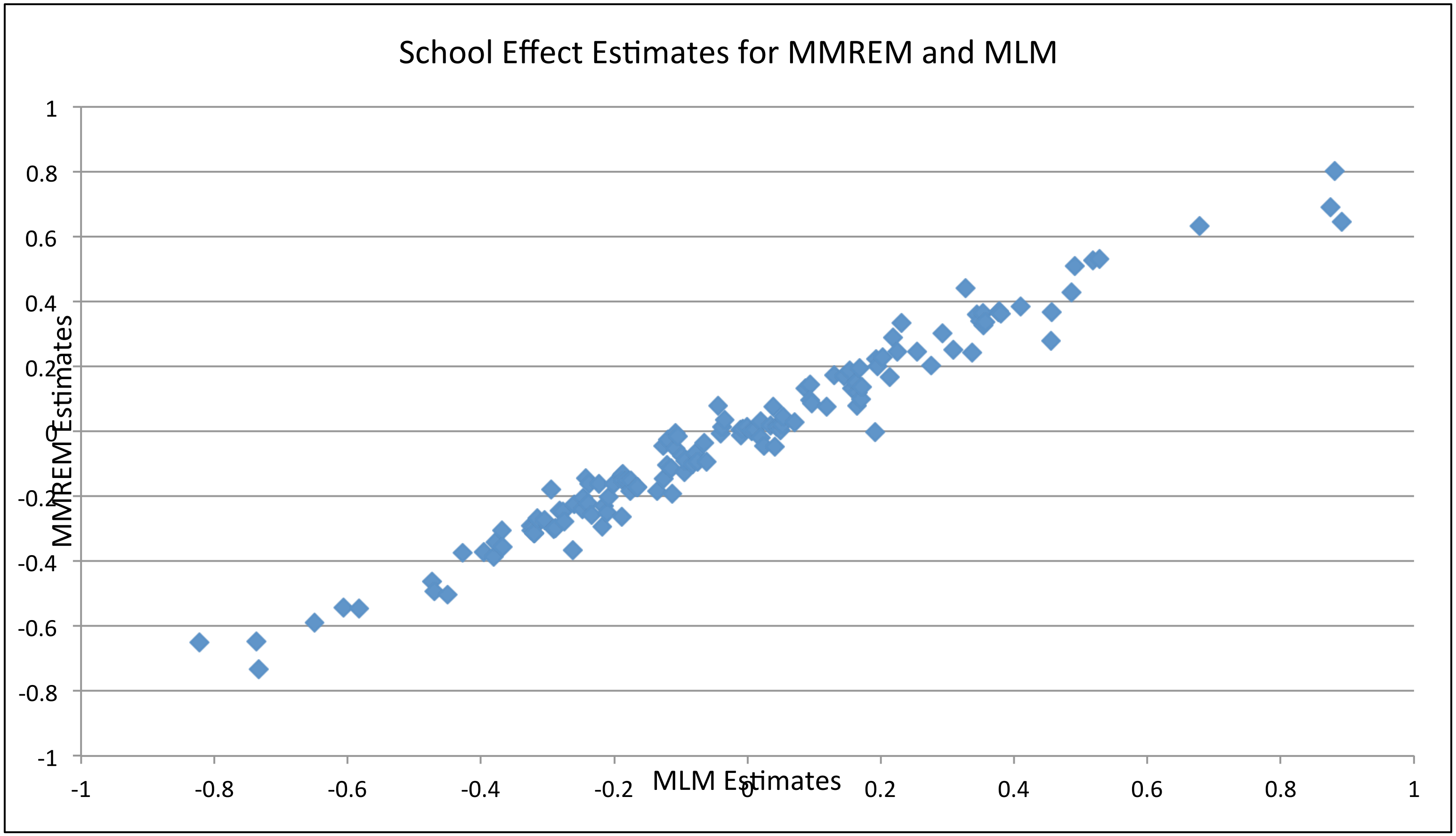
MMREM

- MLM school effect estimates:



MMREM

- School effect estimates from the two models:



MMREM

- In this case, the school effect estimates are highly correlated, *(though not perfectly so).*

- However, the dataset has a low mobility rate.
  - *About 15%*

- Between-model differences in the effect estimates will be greater when the mobility rate is higher.
  - *See Leckie (2009)*

MMREM

- Other examples of multiple membership:

- *Students taught by multiple teachers.*
- *People work at multiple companies.*
- *Patients treated by multiple doctors/ psychologists*
- *Patients treated in multiple hospitals.*
- *Players play for multiple teams.*

Cross-Classified Random Effects Models

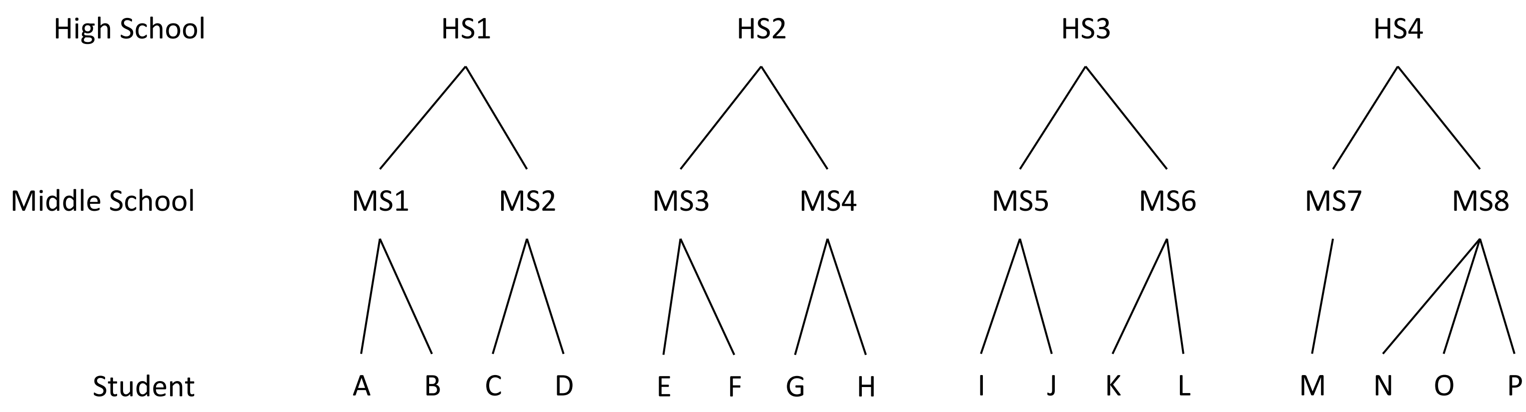
Pure Clustering

vs.

Cross-Classification

Pure Clustering

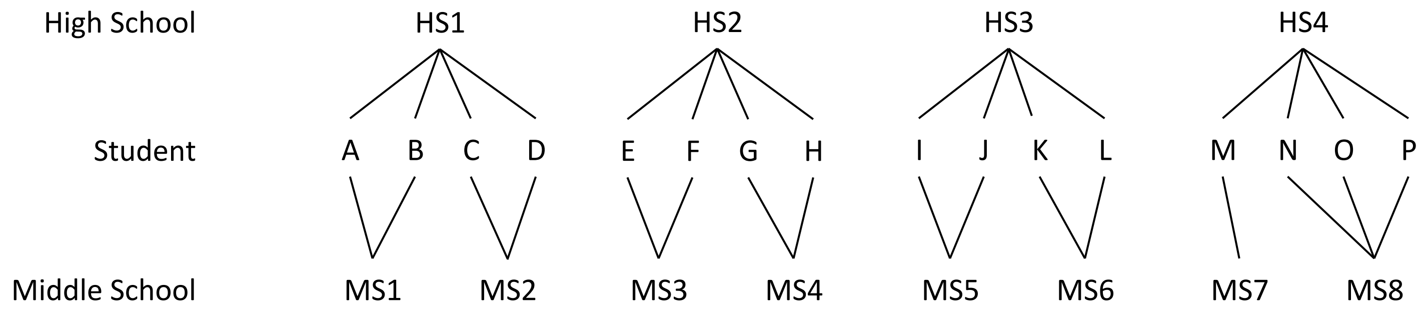
- Students clustered in Middle Schools then High Schools



- This is *pure clustering* of students in middle schools then high schools.

Pure Clustering

- *What makes the clustering pure?*
  - Each student attends a single middle school.
  - Each high school is fed from a fixed set of middle schools – *none of which feed any other high schools*.



Pure Clustering

– Each high school is fed from a fixed set of middle schools – *none of which feed any other high schools*.

	High School							
	1		**2**		**3**		**4**	
Middle School								
1	A	B						
2	C	D						
3			E	F				
4			G	H				
5					I	J		
6					K	L		
7							M	N
8							O	P

Multilevel Models

- With *pure clustering* I can fit the following multilevel model:

$$Y_{ijk} = \gamma_{000} + u_{00k} + r_{0jk} + e_{ijk}$$

- Where *i* indexes the student, *j* indexes the middle school and *k* indexes the high school.

Multilevel Models

- For this three-level multilevel model:

$$Y_{ijk} = \gamma_{000} + u_{00k} + r_{0jk} + e_{ijk}$$

- We assume:

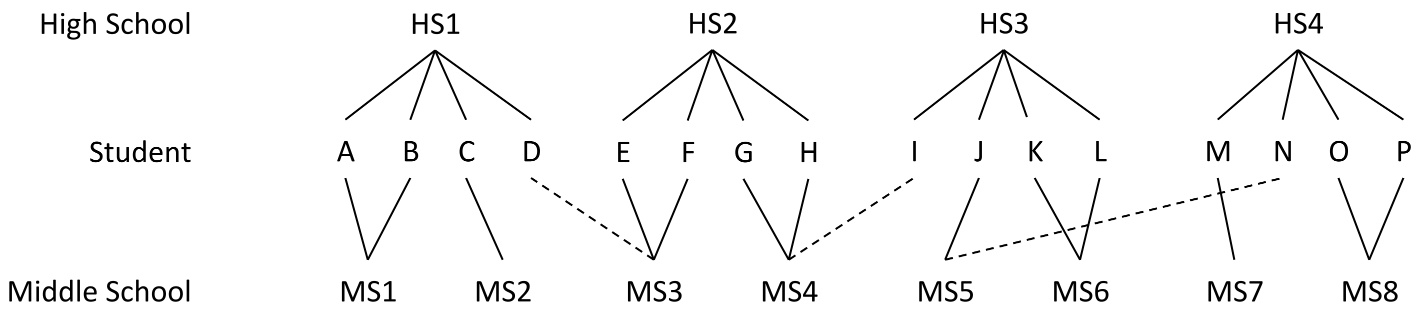
$$u_{00k} \sim N(0, \tau_{u00}) \text{ and } r_{0jk} \sim N(0, \tau_{r00}) \text{ and } e_{ijk} \sim N(0, \sigma^2)$$

- The Intra-class correlations are:

$$ICC_{HS} = \frac{\tau_{u00}}{\tau_{u00} + \tau_{r00} + \sigma^2} \text{ and } ICC_{MS} = \frac{\tau_{r00}}{\tau_{u00} + \tau_{r00} + \sigma^2}$$

Impure Clustering

- *What would make the clustering impure?*
  - Each high school is fed from a set of middle schools – *but some middle schools in that set feed into other high schools.*



Impure Clustering

– Each high school is fed from a set of middle schools – *but some middle schools in that set feed other high schools.*

	High School							
	1		**2**		**3**		**4**	
Middle School								
1	A	B						
2	C							
3		D	E	F				
4			G	H	I			
5						J		N
6					K	L		
7							M	
8							O	P

– Students are *cross-classified* by middle school and high school.

Impure Clustering

- Options for dealing with impurity:
  - *Ignore* Middle School
  - *Ignore* High School
  - *Delete* those who make the clustering impure.
  - Fit a *CCREM*

CCREM

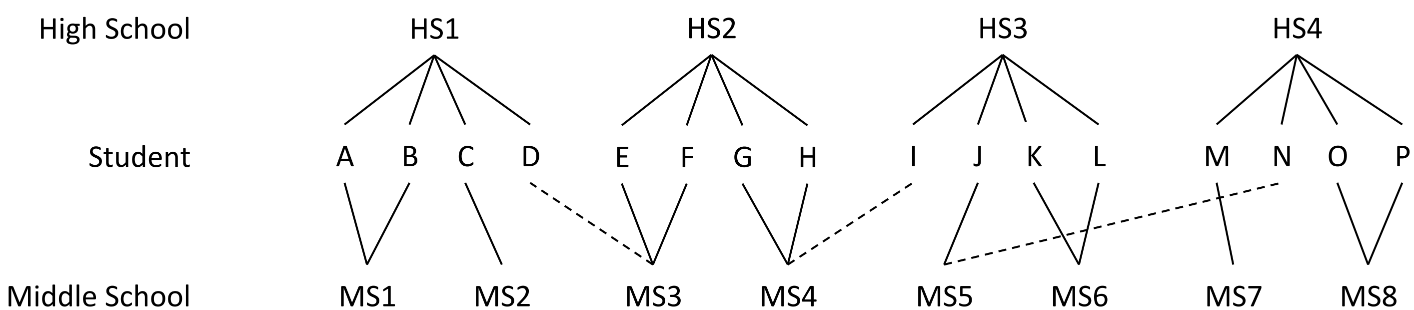
- The CCREM for this scenario can be written as follows:

$$Y_{i(j,k)} = \gamma_{000} + \underbrace{u_{0j0}}_{\text{Middle school effect}} + \underbrace{u_{00k}}_{\text{High school effect}} + e_{i(j,k)}$$

- Where *i* indexes student, *j* indexes middle school and *k* indexes high school.
- Here middle school is classification 1 and high school is classification 2.

CCREM

- Consider student *F* who attended *MS3* and *HS2*:



- His outcome could be modeled as follows:

$$Y_{F(MS3,HS2)} = \gamma_{000} + u_{0MS3,0} + u_{00,HS2} + e_{F(MS3,HS2)}$$

CCREM

- For this CCREM:

$$Y_{i(j,k)} = \gamma_{000} + u_{0j0} + u_{00k} + e_{i(j,k)}$$

- We assume:

$$u_{00k} \sim N(0, \tau_{k00}) \text{ and } u_{0j0} \sim N(0, \tau_{j00}) \text{ and } e_{i(j,k)} \sim N(0, \sigma^2)$$

- The Intra-class correlations are:

$$ICC_{HS} = \frac{\tau_{k00}}{\tau_{k00} + \tau_{j00} + \sigma^2} \text{ and } ICC_{MS} = \frac{\tau_{j00}}{\tau_{k00} + \tau_{j00} + \sigma^2}$$

CCREM Example 1

Estimating Gender and Charter High School Effects

CCREM

Real Data Example

- I have a math test score for 3,435 students.
- All students attended one of 148 middle schools and one of 19 high schools.
- Students are *cross-classified* by middle school and high school.

MMREM

Example data:

Student	Y	MS	HS
1	10	1	1
2	6	2	5
3	5	2	5
4	8	3	8
5	4	3	12

- I have the middle school and high school *ID* for each student.

CCREM

Crosstabs: *Middle School by High School*

```
Columns are levels of MS
Rows are levels of HS
```

Middle school

High school

		1	2	3	4	5	6
1	N	8	0	0	0	53	1
2	N	0	0	0	0	0	0
3	N	0	0	0	0	0	1
4	N	0	0	0	0	0	0
5	N	0	0	3	0	0	52
6	N	0	0	0	1	0	0
7	N	0	7	0	0	0	0
8	N	0	0	0	0	0	0
9	N	45	0	0	6	0	0

CCREM

- I'll start with the following unconditional CCREM:

$$Y_{i(j,k)} = \gamma_{000} + u_{0j0} + u_{00k} + e_{i(j,k)}$$

- Where *i* indexes student, *j* indexes middle school and *k* indexes high school.

CCREM

- Results, (*from MLwiN*):

$$\mathrm{Math}_i \sim \mathrm{N}(XB, \Omega)$$

$$\mathrm{Math}_i = \beta_{0i}\mathrm{CONS}_i$$

$$\beta_{0i} = 5.498(0.186) + u^{(3)}_{0,HS(i)} + u^{(2)}_{0,MS(i)} + e_{0i}$$

$$\left[u^{(3)}_{0,HS(i)}\right] \sim \mathrm{N}(0,\ \Omega^{(3)}_u) \ : \ \Omega^{(3)}_u = \left[0.411(0.216)\right] \longleftarrow \tau_{k00}$$

$$\left[u^{(2)}_{0,MS(i)}\right] \sim \mathrm{N}(0,\ \Omega^{(2)}_u) \ : \ \Omega^{(2)}_u = \left[1.150(0.211)\right] \longleftarrow \tau_{j00}$$

$$\left[e_{0i}\right] \sim \mathrm{N}(0,\ \Omega_e) \ : \ \Omega_e = \left[8.119(0.205)\right] \longleftarrow \sigma^2$$

Deviance(MCMC) = 16940.783(3435 of 3435 cases in use)

CCREM

- The Intra-class correlations are:

$$ICC_{HS} = \frac{\tau_{k00}}{\tau_{k00} + \tau_{j00} + \sigma^2} = \frac{.411}{.411 + 1.15 + 8.119} = .042$$

$$ICC_{MS} = \frac{\tau_{j00}}{\tau_{k00} + \tau_{j00} + \sigma^2} = \frac{1.15}{.411 + 1.15 + 8.119} = .119$$

CCREM

- Suppose I *ignored* middle school and fit a model with students nested in high school.

- This model would be given as follows:

$$Y_{ik} = \gamma_{00} + u_{0k} + e_{ik}$$

- Where *i* indexes student and *k* still indexes high school.
  - *This is just a traditional multilevel model.*

CCREM

- Results:

$$\text{Math}_i \sim \text{N}(XB, \Omega)$$

$$\text{Math}_i = \beta_{0i}\text{CONS}_i$$

$$\beta_{0i} = 5.608(0.166) + u^{(2)}_{0,HS(i)} + e_{0i}$$

$$\left[u^{(2)}_{0,HS(i)}\right] \sim \text{N}(0,\ \Omega^{(2)}_u)\ :\ \Omega^{(2)}_u = \left[0.489(0.210)\right]$$

$$\left[e_{0i}\right] \sim \text{N}(0,\ \Omega_e)\ :\ \Omega_e = \left[8.989(0.219)\right]$$

$Deviance(MCMC)$ = 17291.800(3435 of 3435 cases in use)

Between HS variance

- ICC_{HS} = .489 / (.489 + 8.989) = .052

CCREM

- Ignoring a cross-classified factor typically results in the inappropriate *repartitioning of variance*:

	HS variance	MS variance	Student variance	*DIC*
CCREM	.411	1.15	8.12	16940
MLM with HS only	.489	NA	8.99	17291

- And a worse fitting model.
  - *Higher DIC*

CCREM

- I could add the student-level predictor *Gender* to the CCREM:

$$Y_{i(j,k)} = \gamma_{000} + \gamma_{100} Gender_i + u_{0j0} + u_{00k} + e_{i(j,k)}$$

- Where the *Gender* effect is modeled as fixed across middle schools and high schools

CCREM

- I could also add the high school-level explanatory variable high school charter status, ($Charter_k$).

$$Y_{i(j,k)} = \gamma_{000} + \gamma_{100} Gender_i + \gamma_{001} Charter_k + u_{0j0} + u_{00k} + e_{i(j,k)}$$

Gender effect

HS Charter effect

CCREM

- CCREMs offer many rich modeling possibilities:
  - *Does the gender effect vary across middle schools and high schools?*
  - *Is the gender effect smaller in charter high schools?*
  - *Does the charter high school effect vary across middle schools?*
  - *Is the charter high school effect greater for those who attended non-charter middle schools?*

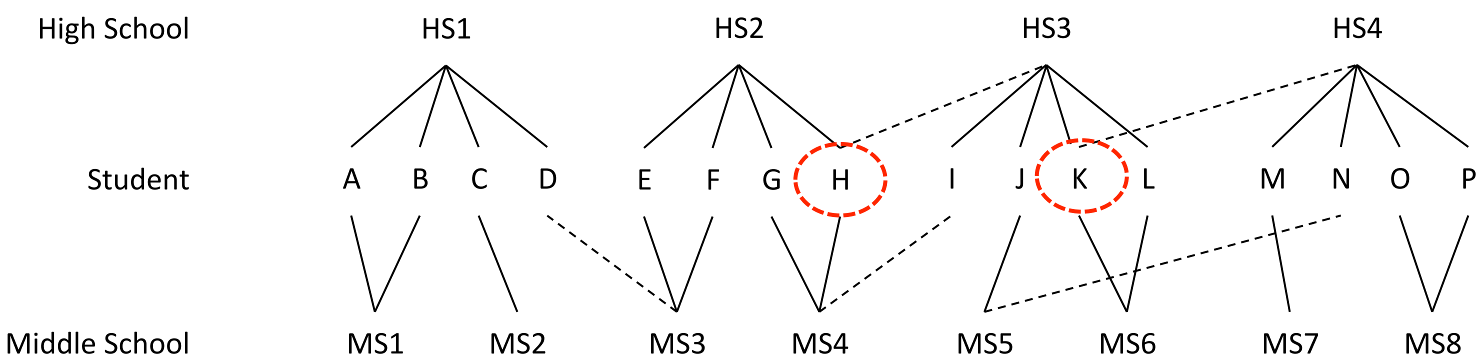
Cross-Classified Multiple Membership Random Effects Models

CCMMREMs

CCMMREM

- Suppose student are *cross-classified* by middle school and high school.

- Also, suppose some students attend *multiple high schools*:



CCMMREM

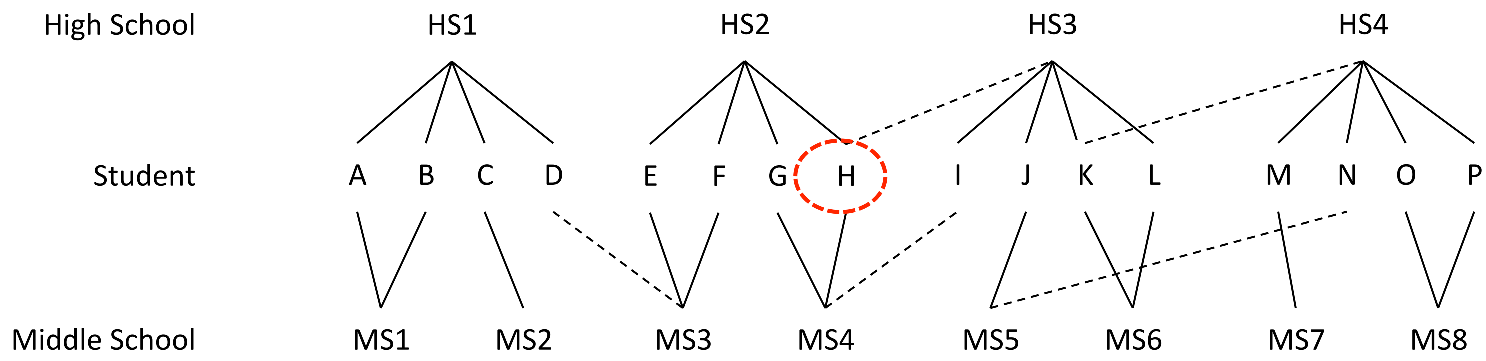
- The CCMMREM for this scenario could be given as follows:

$$Y_{i(j,\{k\})} = \gamma_{000} + u_{0j0} + \sum_{h \in \{k\}} w_{ih} u_{00h} + e_{i(j,\{k\})}$$

- We weight the effects of the high schools that the student attended.

CCMMREM

- The CCMMREM for student H would be:



$$Y_{H(MS4,\{HS2,HS3\})} = \gamma_{000} + u_{0MS4,0}$$

$$+ 0.5 * u_{00,HS2} + 0.5 * u_{00,HS3} + e_{H(MS4,\{HS3,HS4\})}$$

Weighted effect of HS2

Weighted effect of HS3

CCMMREM

- *Example data:*

Student	Y	MS	HS1	HS2	Weight 1	Weight 2
A	100	1	1	1	1	0
…						
H	95	4	2	3	0.5	0.5
…						

- For non-mobile students, the CCMMREM simplifies to a CCREM.

Model Estimation

CCMMREM

- As far as I know:
- *Cross-classified models* can be estimated in:
  - SAS
  - HLM
  - MLwiN
  - WinBUGS
- *Multiple Membership models* can be estimated in:
  - MLwiN, (using MCMC methods)
  - WinBUGS, (using MCMC methods)

References

Beretvas, S.N. (2008). Cross-Classified Random Effects Models. In A.A. O'Connell & D. Betsy McRoach (Eds.), *Multilevel modeling of educational data* (pp. 161-197). Charlotte, SC: Information Age Publishing.

Browne, W.J. (2009) *MCMC Estimation in MLwiN, v2.13*. Centre for Multilevel Modelling, University of Bristol.

Goldstein, H. (2003). *Multilevel statistical models* (3rd ed.). New York: Hodder Arnold.

Goldstein, H., Burgess, S., & McConnell, B. (2007). Modeling the effect of pupil mobility on school differences in educational achievement. *Journal of the Royal Statistical Society, A., 170*(4), 941-954.

Meyers, J. & Beretvas, S.N. (2006). The impact of inappropriate modeling of cross-classified data structures. *Multivariate Behavioral Research, 41,* 473-497.

Raudenbush, S.W. & Bryk, A.S. (2002). *Hierarchical linear models: Applications and data analysis methods.* Thousand Oaks, CA: Sage Publications, Inc.

Thanks

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Email me for example WinBUGS code